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## Tabular and Deep Learning of Whittle Index

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## Introduction

- Whittle index policy is an asymptotically optimal heuristic for solving Restless Multi-Armed Bandit Problems (RMBAP).
- We propose two algorithms, QWI and QWINN, for the computation of such indices.
- Both employ a two time-scale system for the computation of the indices and the Q-values of each state/action.

## Motivation

### Asymptotically Optimal Heuristics for Restless Multi-Armed Bandit Problems (RMABP)

- Load-balancing problems
- Machine maintenance
- Health-care systems

## QWI/QWINN Algorithm

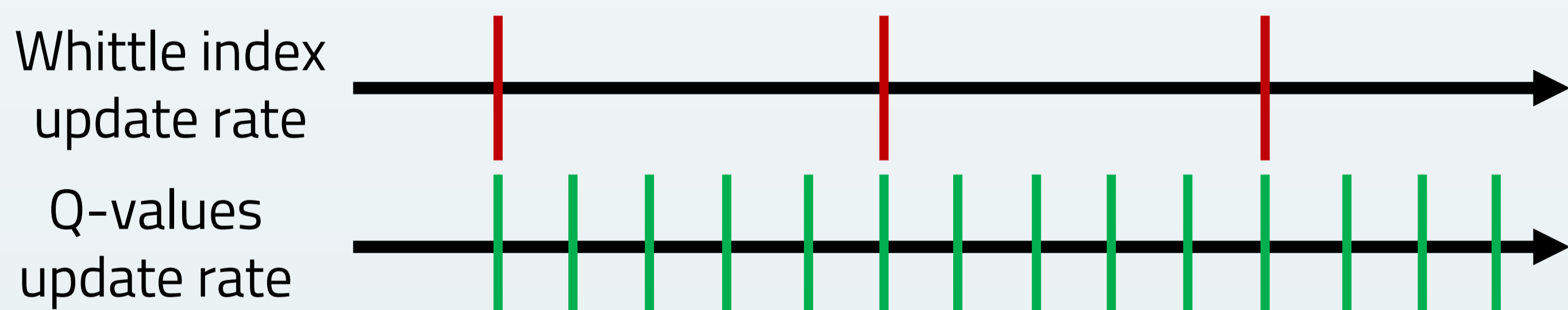
### Algorithm:

Evaluate the **Q-values** for a given state/action  $(s_n, a_n)$  using the index for all states  $x \in \mathcal{S}$

$$\begin{aligned} Q_{n+1}^x(s_n, a_n) &= (1 - \alpha(n))Q_{n+1}^x(s_n, a_n) \\ &+ \alpha(n) \left( (1 - a_n)(r_0(s_n) + \lambda_n(x)) + a_n r_1(s_n) \right) \\ &+ \gamma \max_{v \in \{0,1\}} Q_n^x(s_{n+1}, v) \end{aligned}$$

Update **Whittle index** for all states  $x \in \mathcal{S}$

$$\lambda_{n+1}(x) = \lambda_n(x) + \beta(n) + (Q_n^x(x, 1) - Q_n^x(x, 0))$$



### Two time-scales set through learning rate

- **Fast time-scale:** Q-values

$$\alpha(n): \sum_n \alpha(n) = \infty, \quad \sum_n \alpha(n)^2 < \infty$$

- **Slow time-scale:** Whittle index

$$\beta(n): \sum_n \beta(n) = \infty, \quad \sum_n \beta(n)^2 < \infty, \quad \beta(n) = o(\alpha(n))$$

### Neural Network implementation (QWINN):

- Hybrid system:
  - Neural network to approximate Q-values

$$\begin{aligned} Q_{target}^x(s_n, a_n) &= (1 - a_n)(r_0(s_n) + \lambda_n(x)) + a_n r_1(s_n) \\ &+ \gamma \max_{v \in \mathcal{A}} Q_{\theta'}^x(s_{n+1}, v) \end{aligned}$$

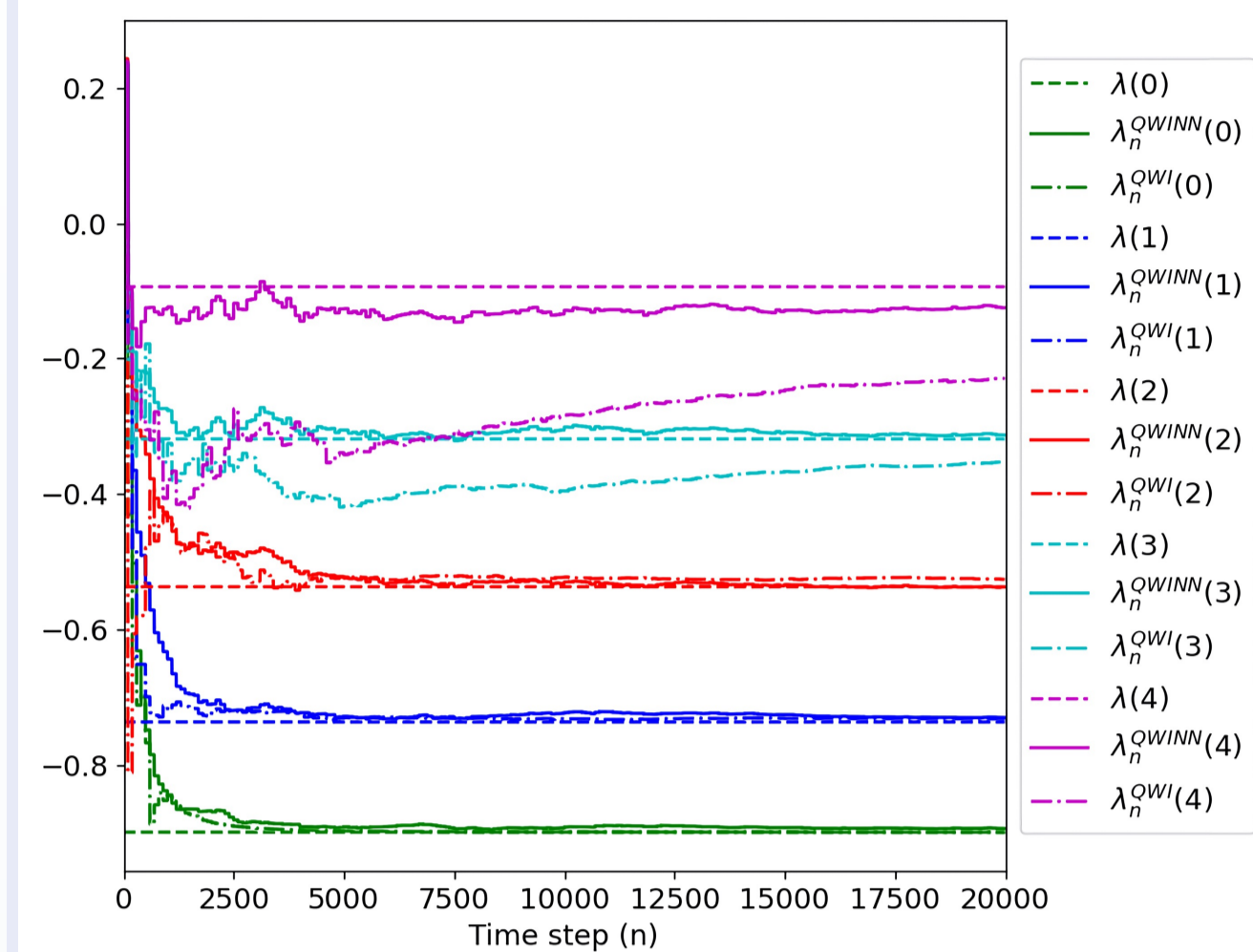
- Tabular computation for Whittle index

$$\lambda_{\theta, n+1}(x) = \lambda_{\theta, n}(x) + \beta(n) \left( Q_{\theta, n}^x(x, 1) - Q_{\theta, n}^x(x, 0) \right)$$

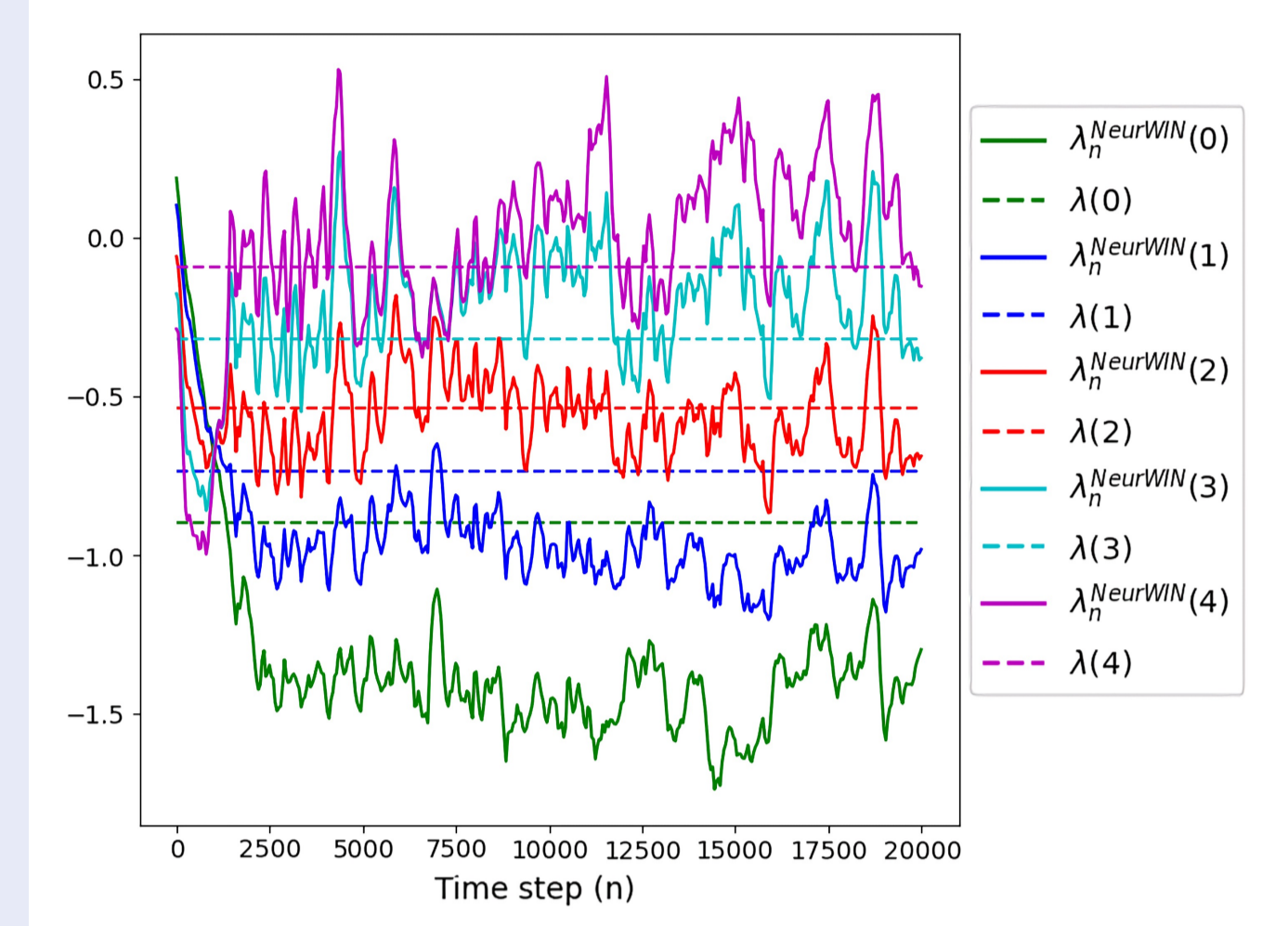
## Results

### Restart problem

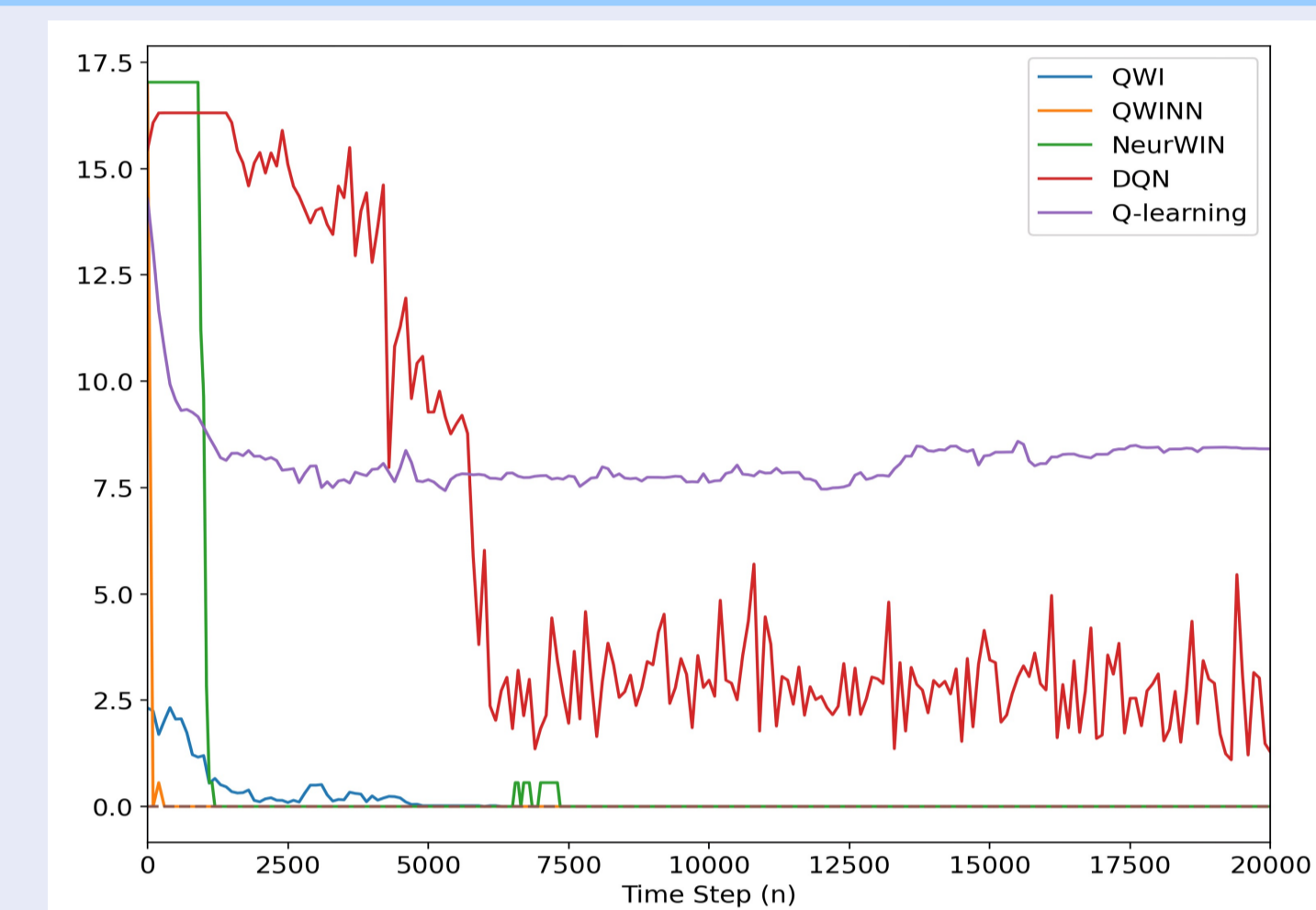
#### QWI/QWINN indices



#### NeurWIN indices [3]

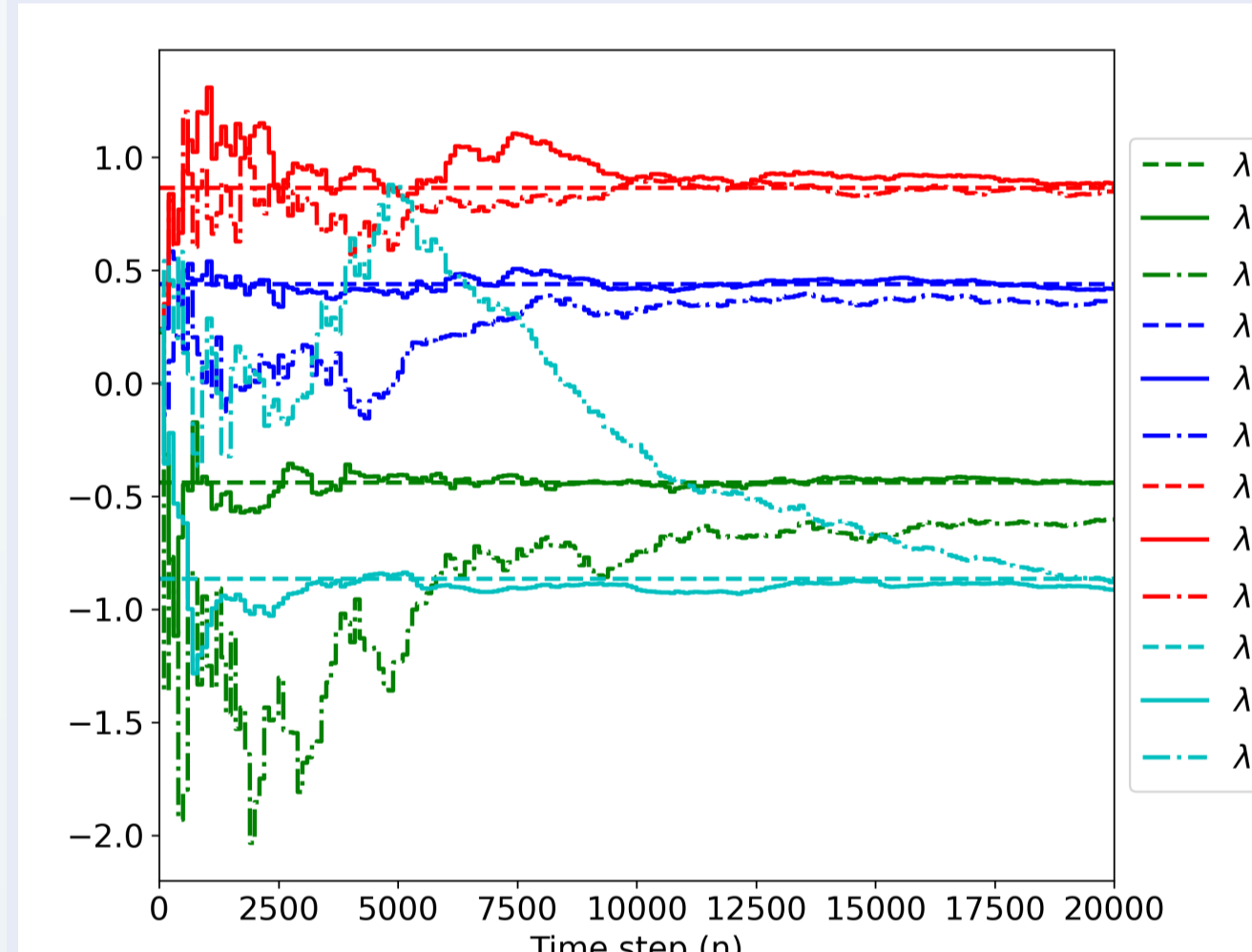


### Bellman Relative Error (lower is better)

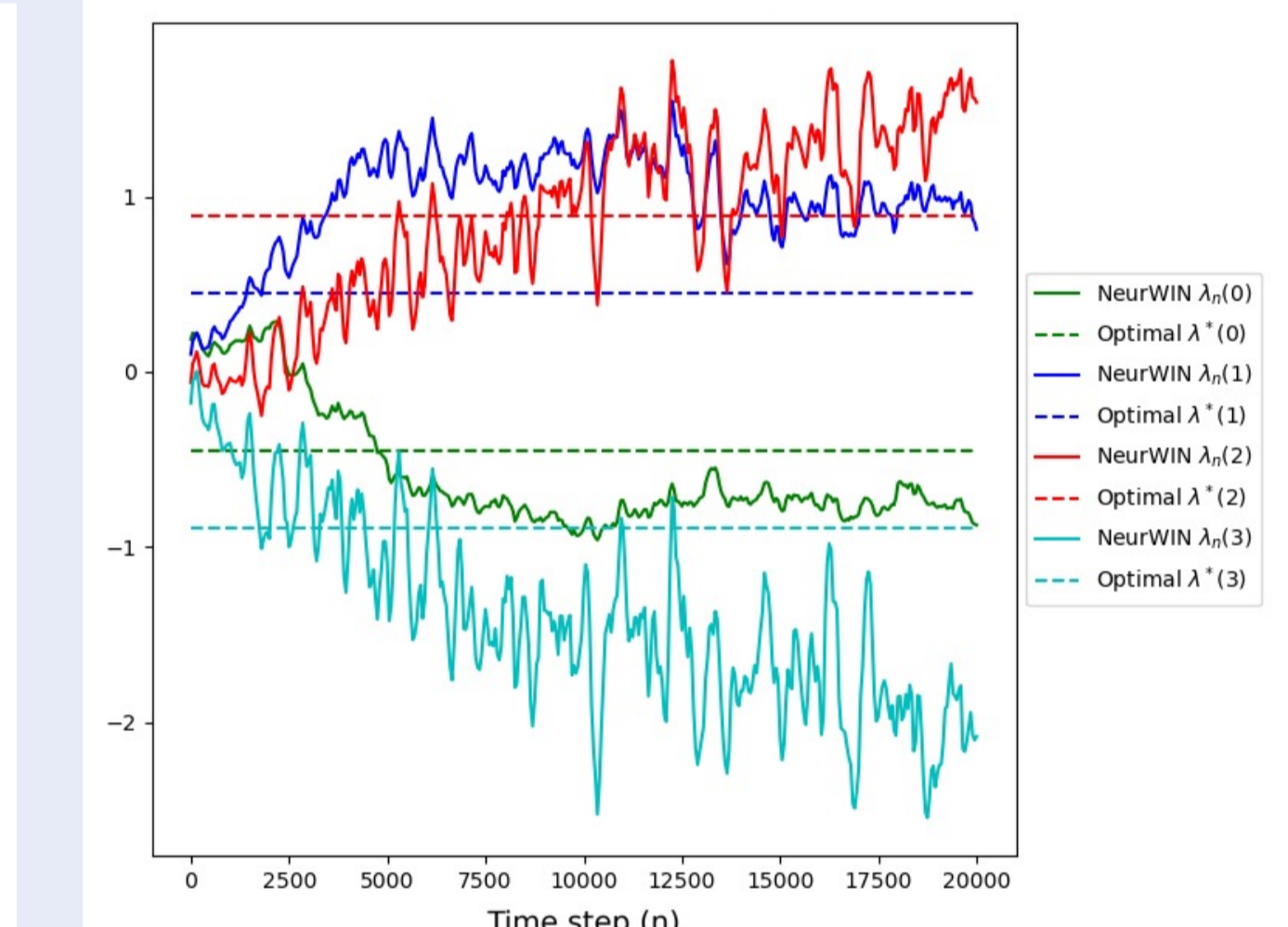


### Circular problem

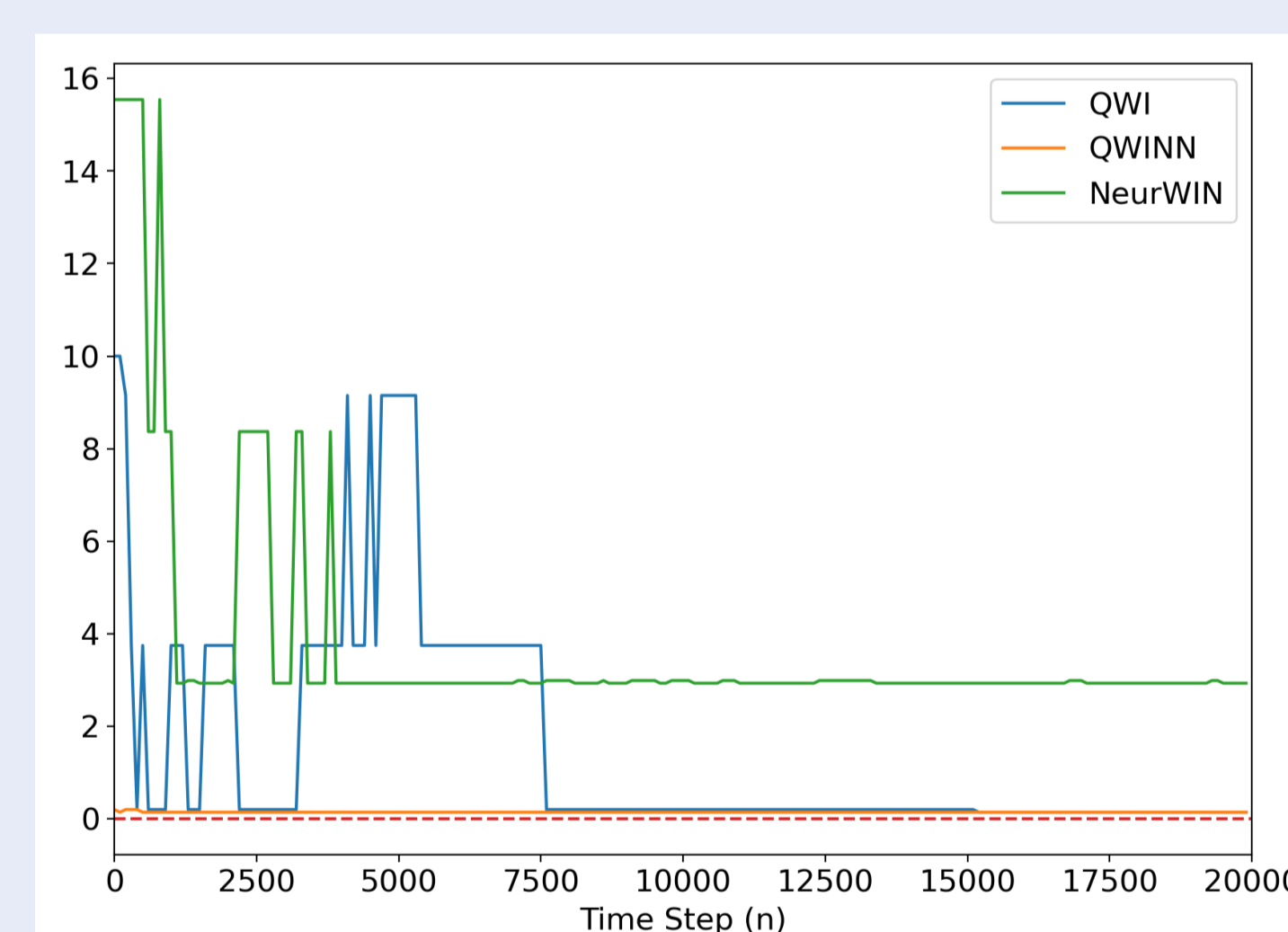
#### QWI/QWINN indices



#### NeurWIN indices [3]



### Bellman Relative Error (lower is better)



## Conclusion

- Two time-scale implementation to decouple Q-learning and Whittle index updates
- Fast and stable convergence to theoretical Whittle index
- Neural Network implementation improves performance in underexplored states

## References

1. Abounadi, Jinane / Bertsekas, Dimitris / Borkar, Vivek S., Learning algorithms for Markov decision processes with average cost, 2001
2. Avrachenkov, Konstantin E. / Borkar, Vivek S., Whittle index based Q-learning for restless bandits with average reward, 2022-05, Automatica, Vol. 139, p. 110186
3. Nakhleh, Khaled / Ganji, Santosh / Hsieh, Ping-Chun / Hou, I.-Hong / Shakkottai, Srinivas, NeurWIN: Neural Whittle Index Network For Restless Bandits Via Deep RL, 2021