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Tabular and Deep Learning of Whittle Index

Francisco Robledo, Vivek Borkar, Urtzi Ayesta, Konstantin Avrachenkov

Introduction

- Whittle index policy is an asymptotically optimal heuristic for solving Restless Multi-Armed Bandit Problems (RMBAP).
- We propose two algorithms, QWI and QWINN, for the computation of such indices.
- Both employ a two-time-scale system for the computation of the indices and the Q-values of each state/action.

Motivation

Asymptotically Optimal Heuristics for Restless Multi-Armed Bandit Problems (RMABP)
- Load-balancing problems
- Machine maintenance
- Health-care systems

QWI/QWINN Algorithm

- **Algorithm:** Evaluate the Q-values for a given state/action \((s_n, a_n)\) using the index for all states \(x \in S\)

\[
Q_{x+1}^\beta(s_n, a_n) = (1 - \alpha(n))Q_{x+1}^\beta(s_n, a_n) + \alpha(n)
\]
\[
+ \left((1 - \alpha(n))(r_0(s_n) + \lambda_n(x)) + a_n r_1(s_n)\right) + \gamma \max_{v \in \mathcal{A}} Q_v^\beta(s_{x+1}, v)
\]

Update **Whittle index** for all states \(x \in S\)

\[
\lambda_{n+1}(x) = \lambda_n(x) + \beta(n) (Q_n^\beta(x, 1) - Q_n^\beta(x, 0))
\]

**Two time-scales** set through learning rate
- **Fast time-scale:** Q-values
  \[\alpha(n): \sum_n \alpha(n) = \infty, \quad \sum_n \alpha(n)^2 < \infty\]
- **Slow time-scale:** Whittle index
  \[\beta(n): \sum_n \beta(n) = \infty, \quad \sum_n \beta(n)^2 < \infty, \quad \beta(n) = o(\alpha(n))\]

Neural Network implementation (QWINN):
- Hybrid system:
  - Neural network to approximate Q-values
    \[
    Q_{\text{Target}}^\beta(s_n, a_n) = (1 - \alpha_n)(r_0(s_n) + \lambda_n(x)) + a_n r_1(s_n)
    \]
    \[
    + \gamma \max_{v \in \mathcal{A}} Q_v^\beta(s_{x+1}, v)
    \]
  - Tabular computation for Whittle index
    \[
    \lambda_{\alpha,n+1}(x) = \lambda_{\alpha,n}(x) + \beta(n) \left(Q_{\alpha,n}^\beta(x, 1) - Q_{\alpha,n}^\beta(x, 0)\right)
    \]

Results

**Restart problem**

<table>
<thead>
<tr>
<th>QWI/QWINN indices</th>
<th>NeurWIN indices [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Circular problem" /></td>
<td><img src="image2.png" alt="Circular problem" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Bellman Relative Error (lower is better)" /></td>
<td><img src="image4.png" alt="Bellman Relative Error (lower is better)" /></td>
</tr>
</tbody>
</table>

**Conclusion**

- Two time-scale implementation to decouple Q-learning and Whittle index updates
- Fast and stable convergence to theoretical Whittle index
- Neural Network implementation improves performance in underexplored states

References


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