



**HAL**  
open science

## Is agglomeration desirable?

Fabien Candau

► **To cite this version:**

| Fabien Candau. Is agglomeration desirable?. 2009. hal-03550799

**HAL Id: hal-03550799**

**<https://univ-pau.hal.science/hal-03550799>**

Preprint submitted on 1 Feb 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Université de Pau  
et des  
Pays de l'Adour

Working Papers Series

CATT WP No. 3.  
October 2009

Is Agglomeration Desirable ?

Fabien Candau

# Is agglomeration desirable?\*

Fabien Candau (Université de Pau et des Pays de l'Adour)<sup>†</sup>

## Abstract

By introducing heterogeneity between high-skilled mobile and low-skill immobile workers in a model with land rent and commuting costs we develop a new model in the field of the new economic geography. This model, which can reveal a dispersion-agglomeration-dispersion configuration when trade gets freer, is used to compare *via* the Pareto criterion the two possible market outcomes, i.e. agglomeration and dispersion. When these equilibria can be ranked it is shown that dispersion can be a Pareto-efficient outcome.

*JEL classification:* F12; R13

*Keywords:* Economic geography; Cities; Trade; Welfare.

## 1 Introduction

Surprisingly in his Nobel lecture, Paul Krugman [2008] challenged the conclusions of the Core-Periphery model (Krugman [1991]) in part because some empirical studies reveal that its main result—agglomeration of activities—appears to belong to the past at least in developed countries<sup>1</sup>. In France, Combes et al. [2008] indicate that manufacturing and services experienced bell-shaped spatial development curves, according to which agglomeration reached maxima around 1930 and then decreased. According to Krugman [2008], such a process of dispersion "may be explained by the decline of increasing returns as a force in the world economy".

This study adopts an alternative point of view, though one already suggested by Krugman and Livas [1996], and argues that agglomeration may destroy itself by increasing the cost of living in big cities. A welfare analysis also serves to help determine the conditions in which agglomeration or dispersion may result in better social outcomes. Such questions generate ongoing and interesting debates, starting with the welfare-based analysis of the Core-Periphery model by Charlot et al. [2006]. These authors demonstrate that agglomeration and dispersion could not be Pareto ranked, because regardless of the value of trade costs, the Core's inhabitants prefer agglomeration, whereas Peripheral workers prefer a dispersed equilibrium. Ottaviano and Robert-Nicoud [2006] and Robert-Nicoud

---

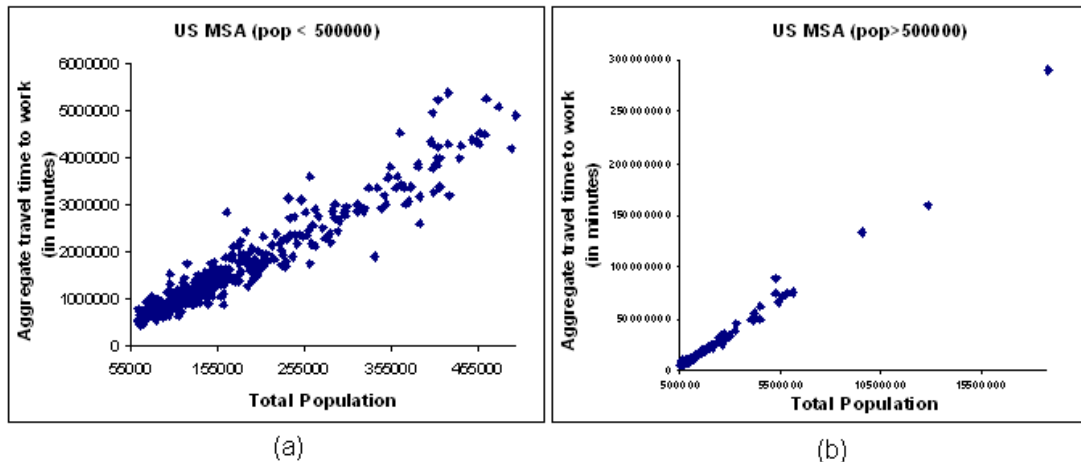
\*I am particularly grateful to Marc Fleurbaey, Carl Gaigné, Frédéric Robert-Nicoud, Sylvie Charlot and Jacques Thisse for helpful comments.

<sup>†</sup>fabien.candau@univ-pau.fr

<sup>1</sup>Krugman particularly notes work by Kim [1998] showing that U.S. manufacturing industries have experienced a dispersion tendency since 1950. For a defence of the relevance of Krugman's contemporary contributions, see Brülhart [2009]. For deeper analysis of his well-deserved Nobel Prize, see Behrens and Robert-Nicoud [2009], Brackman and Garretsen [2009], and Fujita and Thisse [2009].

[2006] further find, using models with vertical links between firms, that agglomeration Pareto-dominates dispersion when trade costs are low enough.<sup>2</sup>

This conclusion may emerge because in these three models, the cost of living is always smaller at the Core than at the Periphery, because goods are produced there, which reduces imports and the burden of trade costs. Yet the cost of moving goods is only a part of the cost of living; for example, urban costs for commuting and land rent also matter. By integrating these features it is doubtful to find that the cost of living is still smaller in big cities since urban costs seem to increase with respect to the urban size. For example, in 1996, the rent per square meter within the city of Paris was 84% higher than the national average (Cavailhès et al. [2004]). Furthermore in US, the aggregate travel time to work (in minutes) in 2007, calculated for more than 500 Metropolitan and Micropolitan Statistical Area (MSA), correlates positively with the population living in such areas, as Figure below reveals.<sup>3</sup>



Therefore, by integrating commuting costs and land rent into the NEG framework, this study suggests that, depending on structural parameters, the cost of living may be higher in the Core than at the Periphery. Other models include these costs as well, such as Krugman and Livas [1996], Tabuchi [1998], Helpman [1998], Sudekum [2006], Murata and Thisse [2006], and Alonso-Villar [2008]. However, the question of the social desirability of agglomeration remains insufficiently addressed. In recent work, Sudekum [2008] and Pflüger and Sudekum [2008.b] build models that share some of the properties proposed herein, including the possibility of a higher cost of living in the Core than at the Periphery. Their model is analytically solvable and features a reasonable welfare analysis; however, they consider a different setting, in which the dispersive force stems from immobile housing

<sup>2</sup>Gagné [2006] instead finds that dispersion can be preferred by everyone in some conditions; Ottaviano and Thisse [2002] further show that agglomeration is not necessarily efficient, even for mobile workers. Pflüger and Sudekum [2008.a] offer a survey of welfare in models in which workers are geographically mobile.

<sup>3</sup>Source: US Census Bureau (American Community Survey). Figure a shows areas with fewer than 500,000 people; Figure b depicts areas with more than 500,000 inhabitants.

stock, and they do not consider income effect for housing, which is questionable from an empirical point of view.<sup>4</sup>

Moreover, their model features no distinct, multiple equilibria; agglomeration occurs gradually. The present study instead allows dispersion and agglomeration to remain stable for the same set of parameters. The analysis therefore provides a complement to previous work, in that it can assess welfare when multiple equilibria are stable. In some circumstances, the market can lead to an inefficient equilibrium if agglomeration occurs, because that spatial configuration harms Peripheral workers while dispersion is also possible and benefits everyone in terms of welfare.

Therefore, this study reveals that in some specific conditions, decentralization policies can be beneficial if agglomeration occurs, whereas agglomeration policies are not recommended according to the Pareto criterion if dispersion emerges spontaneously.

The remainder of this article is structured as follows: the first part presents the economic geography model used to gather the analytical results. Next, the basic results demonstrate that the model can reveal a dispersion—agglomeration—dispersion configuration when trade becomes freer. The final section integrates welfare into the analysis.

## 2 The model

Consider an economy with two regions (labelled  $r=1,2$ ) and two sectors. Each region is formed by a city spread along a one-dimensional space  $X$ . The amount of land available at each location  $x \in X$  is equal to one. The background sector  $A$  (agriculture) produces a homogenous good under constant returns to scale (or CRS) in a perfectly competitive environment using unskilled workers (whose wages are denoted by  $w_A$ ) only. This good is taken as the numeraire, its output is freely traded; so that its price is 1 in both regions. This yields  $w_A = 1$  because of CRS. The manufacturing sector  $M$  produces a differentiated product under increasing returns to scale (or IRS) in a monopolistically competitive environment *à la* Dixit and Stiglitz [1977] using skilled and unskilled workers<sup>5</sup>. We assume that skilled workers (also called entrepreneurs in the literature) are mobile from one city to the next while unskilled workers are spatially immobile but perfectly mobile between sectors. Unlike unskilled who do not need to commute and who are located in city suburbs, where land rent is null, skilled workers who own one unit of land are spread along a line, and because their business is located in the middle of this line (called the Central Business District (CBD)) they need to commute. Indeed all industrial firms located in region  $r$  are set up at the CBD situated at the origin  $x = 0$  of  $X$ . These commuting costs have a direct impact on their labour force. Each skilled consumes one unit of land, supplies one unit of labor, and commutes to the CBD. Hence, in equilibrium, skilled are equally distributed around the CBD of region  $r$  whose urban landscape is therefore given by  $[-h_r/2, h_r/2]$  with  $h_r$  the number of skilled workers in region  $r$ . Commuting costs have an iceberg form, thus implying that the effective labor supply by a skilled worker living at a distance  $|x|$  from the CBD is given by:

$$s(x) = 1 - 2\theta |x| \quad \text{with } x \in [-h_r/2, h_r/2] \quad (1)$$

<sup>4</sup>As Sudekum [2008] notices "income elasticities for housing are positive in reality".

<sup>5</sup>The model thus differs from Krugman and Livas [1996], Murata and Thisse [2006] and Helpman [1998] since these authors do not introduce the agricultural sector and consider only one kind of workers.

where  $\theta$  (with  $\theta < 1$ ) is skilled worker' commuting cost,  $|x|$  measures distance to CBD. Indeed as the number of skilled in one city is  $h_r$ , the maximal distance from the CBD is  $\frac{h_r}{2}$ , thus the total labour supply net of commuting cost in one city is equal to :

$$S_r = \int_{-h_r/2}^{h_r/2} s(x)dx = h_r(1 - \theta h_r/2). \quad (2)$$

As land rent at both edges of the segment is normalized to zero, if  $w_r$  is the wage of skilled near the CBD, then wage net of commuting costs earned at both edges is:

$$s(h_r/2)w_r = s(-h_r/2)w_r = (1 - \theta h_r)w_r. \quad (3)$$

Because consumers are identical in terms of preferences and income, at equilibrium they must reach the same utility level. Thus skilled workers who live on the fringe of the segment only receive a net wage of  $(1 - \theta h_r)w_r$  but pay no land rent. On the contrary, workers who live near the CBD do not pay significant commuting costs, but the price of the services yielded by land is higher in this location. Thus, the increase in nominal wage near central places offsets land rent. A move from the suburb to the CBD implies a decrease in commuting and therefore an increase in net wage, but also an equivalent increase in land rent which equalizes utility among individuals. In other terms, the following condition must be verified:

$$s(x)w_r - R_r(x) = (1 - \theta h_r)w_r,$$

where  $s(x)$  is the total amount supplied by a skilled worker who lives on the fringe of the CBD,  $R_r(x)$  is the land rent prevailing at  $x$  in region  $r$ , while the right-hand side represents the wage net of commuting costs earned at both edges given by (3). By inserting expression (1) into this system we find the following land rent:

$$R_r(x) = \theta(h_r - 2|x|)w_r \quad \text{with } x \in [-h_r/2, h_r/2].$$

From this equation we can find the Aggregate Land Rent (ALR):

$$ALR_r = \int_{-h_r/2}^{h_r/2} R_r(x)dx = \frac{\theta h_r^2 w_r}{2}.$$

While on the one hand, Tabuchi [1998] assumes that there are absentee landlords, and on the other, Helpman [1998] assumes that the aggregate land rent is owned at the global level, here it is assumed with Krugman and Livas [1996] that each skilled worker owns an equal share of the ALR where they reside. Thus their non salaried income is:

$$\frac{ALR_r}{h_r} = \frac{\theta h_r w_r}{2}. \quad (4)$$

We can now turn to consumers' behavior. All consumers share the same Cobb-Douglas utility function and consume one industrial good, which is a composite of different varieties,

and one agricultural good:

$$U_r = M_r^\mu A_r^{1-\mu} \quad \text{with} \quad M_r = \left[ \int_{i \in n_r} m_{rr}(i)^{\frac{\sigma-1}{\sigma}} di + \int_{i \in n_s} m_{rs}(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \quad (5)$$

where  $M_r$  is the consumption of a manufactures aggregate,  $A_r$  of the agricultural good,  $n_r$  is the set of varieties produced in region  $r$  and  $n_s$  varieties produced in region  $s$  ( $r, s = 1, 2$  with  $r \neq s$ ),  $\sigma > 1$  is the elasticity of substitution among these varieties. A share  $\mu$  of nominal income, denoted  $Y_r$ , is spent on manufactures, and  $1 - \mu$  on agricultural produce. Industrial varieties are exchanged between regions under transaction costs which take the form of iceberg costs:  $\tau > 1$  units of the variety must be sent from the origin for one unit to arrive at destination. The budget constraint is then given by:

$$\int_{i \in n_r} p_r(i) m_{rr}(i) di + \int_{i \in n_s} p_s(i) \tau m_{rs}(i) di + p_a A = Y_r,$$

where  $p_a$  is the price of the agricultural good and with  $Y_r$  given by:

$$Y_r = h_r \left(1 - \frac{\theta}{2} h_r\right) w_r + L_r \quad (6)$$

where  $(1 - \frac{\theta}{2} h_r) w_r$  comes from the income of land ownership ( $\theta h_r w_r / 2$ ) and from the wage net of commuting costs earned at both edges ( $(1 - \theta h_r) w_r$ ) and where  $L_r$  represents the number of unskilled in region  $r=1,2$ . In the following we consider that the population of unskilled are identical in region 1 and 2, i.e  $L_1 = L_2 = L$ .

Then the solution to the utility maximization problem generates the following demands in  $r$  for a typical variety produced respectively in location  $r$  and  $s$ :

$$\begin{aligned} m_{rr}(i) &= \mu \frac{Y_r}{P_r^{1-\sigma}} p_r(i)^{-\sigma} \\ m_{rs}(i) &= \mu \frac{Y_r}{P_r^{1-\sigma}} p_s(i)^{-\sigma} \tau^{-\sigma} \end{aligned}$$

with  $P_r$  the price index in  $r$ :

$$P_r = \left[ \int_{i \in n_r} p_r(i)^{1-\sigma} di + \int_{i \in n_s} p_s(i)^{1-\sigma} \tau^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}, \quad (7)$$

We can now turn to the firms' behavior. Concerning the cost function, we assume that the production of a typical variety of manufactured goods involves skilled' services as a fixed cost, and the use of  $\beta$  units of unskilled workers for each unit of output produced. Thus the total cost of producing  $q_r(i)$  units of a typical manufactured variety is:

$$TC_r(i) = w_r + \beta q_r(i), \quad (8)$$

This cost function has been used for the first time by Forslid and Ottaviano [2003] in Krugman's Core-Periphery model and has rarely been applied to a model with land rent and commuting costs, certainly because models which integrate these urban economics features have dropped unskilled workers (see Krugman and Livas [1996], Candau [2008 b,c] or Murata and Thisse [2005]). Such a cost function which is in essence an assumption of convenience turns out to be justifiable on empirical grounds, indeed we implicitly assume that skilled workers perform service tasks in the CBD and that production is being conducted in suburban areas (because unskilled/immobile workers are located there).

Because each firm produces a distinct variety, the number of firms is also the number of varieties consumed. Thus each firm is a monopolist on the production of its variety and maximizes its profit with:

$$q_r(i) = m_{rr}(i) + \tau m_{sr}(i) \quad (9)$$

According to the Dixit-Stiglitz monopolistic competition, a typical firm sets the following price:

$$p_r(i) = \beta\sigma/(\sigma - 1). \quad (10)$$

Prices are thus constant and independent of the residual claimants to the profits earned by skilled workers. Until the end of the article the input-output coefficient is normalized to the reverse of the mark-up:  $\beta = (\sigma - 1)/\sigma$ . With this standard assumption, prices are equal to one where the variety is produced (and to  $\tau$  when the variety is imported).

Under free entry, profits are always equal to zero, which, using (8) and (10), gives the level of output:

$$q_r(i) = \sigma w_r. \quad (11)$$

In equilibrium, a typical firm employs one skilled worker, so that the total demand is  $n$ . As skilled labour supply is exactly  $S$ , the equalization gives the number of varieties produced:

$$n_r = S_r. \quad (12)$$

The number of varieties produced is then proportional to the entrepreneurial force. This equation is important since it embodies increasing returns at the level of the firm and shows that the more symmetric the spatial distribution of skilled workers, the larger the total mass of varieties in the economy. Indeed by inserting  $S_1$  given by (2) in (12) and by differentiating the total mass of varieties with respect to  $h_1$  (the sum of the population is normalized to one:  $h_1 + h_2 = 1$ ), we get  $\partial(n_1 + n_2)/\partial h_1 = \partial(S_1 + S_2)/\partial h_1 = \theta(1 - 2h_1)$  and  $\partial^2(n_1 + n_2)/\partial h_1^2 = \partial^2(S_1 + S_2)/\partial h_1^2 = -2\theta < 0$ . Thus, the number of varieties is maximized at  $h_1 = 1/2$  and declines as  $h_1$  increases. This result corresponds to proposition 1 of Murata and Thisse [2005].

By equalizing the demand (eq.(9)) to the supply (eq.(11)) nominal wages are obtained:

$$w_1 = \frac{bL(bS_2(1 - \phi^2) - (\Delta_2 + \Delta_1\phi))}{b(S_2\Delta_1 + S_1\Delta_2) + b^2S_1S_2(\phi^2 - 1) - \Delta_1\Delta_2} \quad (13)$$

$$w_2 = \frac{bL(bS_1(1 - \phi^2) - (\Delta_1 + \Delta_2\phi))}{b(S_2\Delta_1 + S_1\Delta_2) + b^2S_1S_2(\phi^2 - 1) - \Delta_1\Delta_2} \quad (14)$$

with  $b = \mu/\sigma$  and where  $\Delta_1$  and  $\Delta_2$  are given by:



$$\Delta_1 \equiv P_1^{1-\sigma} = S_1 + \phi S_2, \quad \Delta_2 \equiv (P_2)^{1-\sigma} = \phi S_1 + S_2 \quad (15)$$

From these equations we can see that two opposite forces drive relative nominal wages : on the one hand an increase of skilled workers in one city exacerbates local competition among firms, thus new entry triggers a slump in the price index, and thereby in operating profits too, so that in order to stay in the market firms need to reduce nominal wages (market crowding effect). But on the other hand, as the income generated by the new workers is spent locally, sales and operating profits increase and under the ‘zero profit condition’ this implies a higher nominal wage (the market access effect). However, skilled workers do not consider the relative nominal wage when they decide to migrate but the relative real wage. Hence in the long run, migration stops when real wages are equalized in case of symmetry ( $h_1 = \frac{1}{2}$ ), or when agglomeration in one city generates a higher relative real wage. More precisely it is assumed that migration is regulated by a simple marshallian adjustment:

$$\dot{h} = \gamma_h (V_1 - V_2),$$

where  $\gamma_h$  is the adjustment speed. Then by denoting  $\Omega$  the relative real wage, and defining it by:

$$\Omega = \frac{V_1}{V_2} \quad (16)$$

$$= \frac{w_1}{w_2} \frac{1 - \theta h_1/2}{1 - \theta h_2/2} \left( \frac{\Delta_2}{\Delta_1} \right)^{-a} \quad (17)$$

$$\text{with } a = \frac{\mu}{\sigma - 1}, \quad (18)$$

where  $V_1$  is total real income in region 1, including landowner’ income. There is a stable total agglomeration in region 1 if  $\Omega|_{h_1=1} \geq 1$ , a stable total agglomeration in region 2 if  $\Omega|_{h_1=0} \leq 1$  and a stable dispersed equilibrium if  $\left. \frac{d\Omega}{dh_1} \right|_{h_1=1/2} < 0$ . In the following we are going to analyse under which conditions the symmetric interior equilibria and the Core-Periphery equilibria are stable (while simulations will indicate that other asymmetric interior equilibria are unstable).

Let us notice that in equation (17) two additional forces appear : on the one hand the term  $(1 - \theta h_1/2)$  which enters multiplicatively in the indirect utility, creates a dispersive force independently of trade costs, which is the land market-crowding effect. On the other hand the third term  $\Delta_2/\Delta_1$  is an agglomerative force (price index effect), indeed goods are cheaper in the agglomerated area because imports are lower and thus the burden of trade costs too.

### 3 Spatial equilibria

It is now possible to analyse the location decision of skilled workers. From (15), (13), (14) and (17) the relative real wage  $\Omega$  that determines migration can be written as follows:

$$\Omega = \frac{1 - \theta h_1/2}{1 - \theta h_2/2} \frac{2S_1\phi + S_2(1 + \phi^2 - (1 - \phi^2)b)}{2S_2\phi + S_1(1 + \phi^2 - (1 - \phi^2)b)} \left( \frac{\phi S_1 + S_2}{S_1 + \phi S_2} \right)^{-a} \quad (19)$$

where  $S_1$  and  $S_2$  are given by (2):

$$S_1 = h_1(1 - \theta h_1/2), S_2 = h_2(1 - \theta h_2/2)$$

$$\text{with } h_2 = 1 - h_1$$

Figure 1.a<sup>6</sup> plots  $\Omega$  with respect to  $h_1$  for three different values of trade costs.

As it has been noticed previously  $\Omega$  is driven by four effects: the market access effect and the market crowding effect, which influence nominal wages, and the price index effect and the land market crowding effect, which determine the cost of living. Separate analyses of the relative nominal wage ( $w_1/w_2$  represented by normal lines in Figure 1.b,c,d) and the relative cost of living in region 1 ( $1/(\frac{1-\theta h_1/2}{1-\theta h_2/2}(\frac{\Delta_2}{\Delta_1})^{-a}$  represented by dashed lines in Figure 1.b,c,d) offer more intuitive results.

For example, Figures 1.a and 1.b indicate that when trade costs are relatively high, the dispersed equilibrium is stable. Indeed from dispersion ( $h_1 = 1/2$ ), migrations entail a reduction in the relative nominal wages, as well as in the relative cost of living. But because the decrease in nominal wage is greater than the decrease in the cost of living though (see the solid and dashed lines in Figure 1.b), skilled workers regret their moves and increase their relative real income returning to a dispersed situation (see Figure 1.a for  $\phi = 0.56$ ). When trade is liberalized though (Fig 1.c and Fig 1.a with  $\phi = 0.59$ ), agglomeration emerges as another stable equilibrium. From the dispersed equilibrium, a small shock of migration (e.g., from  $h_1 = 0.5$  to  $h_2 = 0.6$ ) has the same effect, whereas a higher shock ( $h_1 = 0.5$  to  $h_1 = 0.9$ ) reduces the relative cost of living enough to overtake the reduction in the relative nominal wage (see Figure 1.c). This situation attracts other skilled workers, leading to total agglomeration. Finally, in Figure 1.d, the decrease in the relative cost of living is always higher than the decrease in the relative nominal wage, which indicates that agglomeration in regions 1 and 2 are stable, but dispersion is unstable.

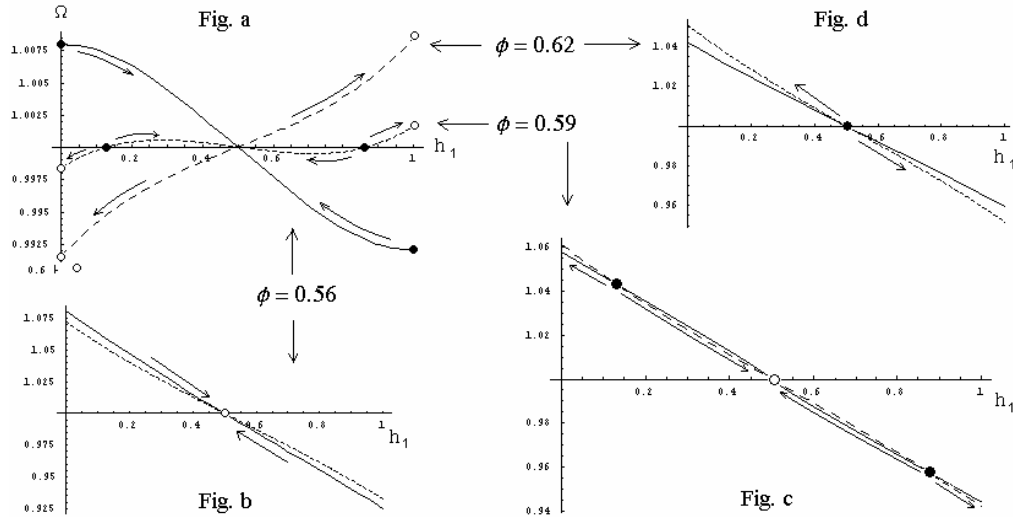


Figure 1

<sup>6</sup>Parameters used in Figures 1 and 2 are  $\mu = 0.6, \sigma = 4, L = 0.5$ .

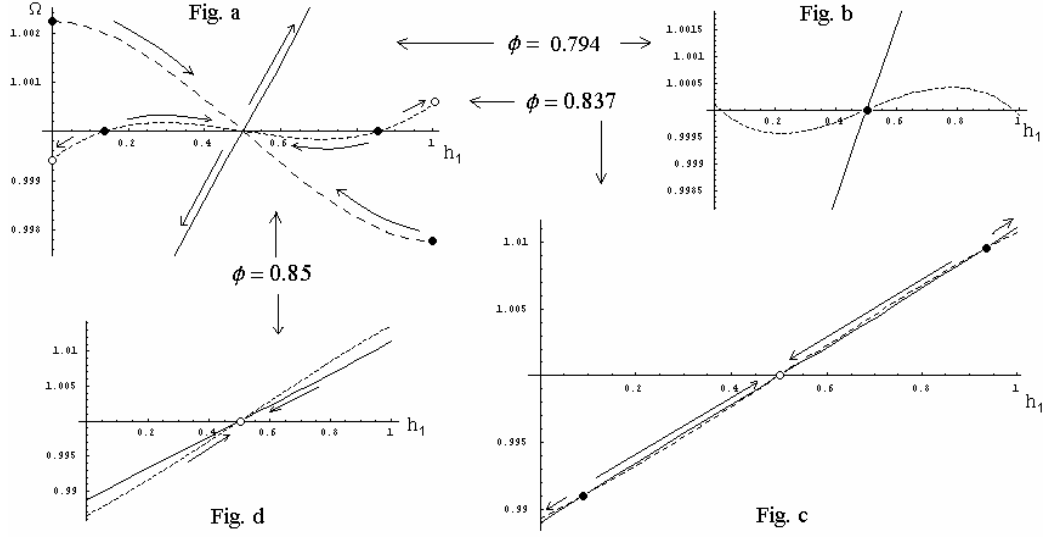


Figure 2

When trade is liberalized even more (e.g. Figure 2.b), migration from  $h_1 = 1/2$  prompts an increase in the relative nominal wage. This change (see the solid lines in Figures 1 and 2) occurs because the market access effect, which previously was dominated by the market crowding effect, becomes the strongest force.

The critical level of trade freedom for which the two effects exactly offset each other can be obtained analytically<sup>7</sup>. Differentiating (17) with respect to  $h_1$  and evaluating it at  $h_1 = 1/2$  shows that the region with more skilled workers offers a higher (lower) wage in the manufacturing sector whenever  $\phi$  is larger (smaller) than the threshold:

$$\phi^w \equiv \frac{1 - b}{1 + b} \tag{20}$$

Another difference between Figure 1 and 2 pertains to the cost of living, which can be higher in the Core than at the Periphery and takes a “S shape” when the number of skilled workers increases.

In Krugman [1991] the former point is never verified, the cost of living is always higher in the Periphery than in the Core. Yet when the cost of living encompasses the price index effect as well as land market crowding effect, at a critical value, agglomerative forces get overtaken. Figures 1.b–d and Figure 2.b depict something similar to Krugman’s [1991] scenario, because the cost of living is higher at the Periphery than in the Core<sup>8</sup>. But with a deeper trade integration (Figures 2.c,d), the cost of living in the Core becomes the highest. Equalizing  $\frac{1-\theta h_1/2}{\Delta_1^{-a}}$  to  $\frac{1-\theta h_2/2}{\Delta_2^{-a}}$  at  $h_1 = 1$  shows that the Periphery offers a lower

<sup>7</sup>This critical point is identical of that one obtained by Forslid and Ottaviano [2002].

<sup>8</sup>Respectively at  $h_1 = 1$  (at  $h_1 = 0$ ) the cost of living in region 1 (region 2) is smaller than the cost of living in the region 2 (region 1).

(higher) cost of living than the Core whenever  $\phi$  is larger (smaller) than the threshold:

$$\phi^c \equiv \left( \frac{(2-\theta)f}{2} \right)^{1/a} \quad (21)$$

Finally, the particular shape of the cost of living curve in Figure 2.b indicates that in a dispersed situation, this cost first increases and then decreases as workers migrates from region 2 to region 1. This shift affects the way redispersion occurs. Specifically, trade liberalization moves the cost of living counterclockwise but also makes (at Figure 2.c) the marginal impact of  $h_1$  on this cost decrease.

Therefore, the relative cost of living is higher than the relative nominal wage when  $h_1$  deviates slightly from dispersion, which restores the stability of the dispersed equilibrium. But when the deviation is higher, the reverse situation occurs, and agglomeration in regions 1 and 2 is still stable (Fig a and c with  $\phi = 0.837$ ). The shape of the cost of living curve then represents a source of a possibly catastrophic redispersion. A small decrease in trade costs (Figure 2.d) causes a sudden dispersion of the industry, such that dispersion becomes the only stable outcome. This catastrophic dispersion also appears in Murata and Thisse's [2005] model, which they argue differs from Helpman's [1998] model, which instead displays a gradual redispersion of activities.

Obviously simulations can provide only a sense of the possible equilibrium outcomes, not the complete picture. Therefore, it is theoretically important to obtain analytical results; this study therefore provides a public policy analysis.<sup>9</sup>

### 3.1 When is the symmetric equilibrium broken?

We start with the symmetric configuration. Clearly the symmetric equilibrium is a persistent steady state (indeed  $\frac{w_1}{w_2} \Big|_{h_1=1/2} = \frac{1-\theta h_1/2}{1-\theta h_2/2} \Big|_{h_1=1/2} = \frac{\Delta_2}{\Delta_1} \Big|_{h_1=1/2} = 1$  thus  $\Omega|_{h_1=1/2} = 1$ ). To study its stability Appendix A displays the analysis of the sign of  $\frac{\partial \Omega}{\partial h_1} \Big|_{h_1=1/2}$  and allows to prove the following result.

**Proposition 1** *The symmetric equilibrium is stable if and only if:*

$$\Lambda(\phi) \equiv \Lambda_A(a + \Lambda_B) < \frac{\theta/2}{2-\theta} \equiv \Gamma(\theta) \quad (22)$$

with  $\Lambda_A = \frac{1-\phi}{1+\phi}$  and  $\Lambda_B = -\frac{1-b-\phi(1+b)}{1-b+\phi(1+b)}$ .

**Proof.** See Appendix A. ■

Since  $\frac{\partial \Lambda_A}{\partial \phi} = -\frac{2}{(1+\phi)^2} < 0$  while  $\frac{\partial(a+\Lambda_B)}{\partial \phi} = \frac{2-2b^2}{(1+b(\phi-1)+\phi)^2} > 0$  we can deduce that  $\Lambda(\phi)$  can be bell-shaped and then the condition  $\Lambda(\phi) = \Gamma(\theta)$  can admit at most two solutions. Indeed by resolving  $\Lambda(\phi) < \Gamma(\theta)$  we find that the symmetric equilibrium is stable if  $\phi < \underline{\phi}^b$

<sup>9</sup>Moreover, such results are important from an empirical point of view, because some research directly tests them (e.g., Head and Mayer [2004]).

and/or if  $\phi > \overline{\phi^b}$  with  $\underline{\phi^b}$  and  $\overline{\phi^b}$  given by:

$$\begin{aligned}\underline{\phi^b} &= \frac{A + \theta + \sqrt{B^2 + 4\theta A}}{B + 2ab(\theta - 2) + (\theta - 4)}, \\ \overline{\phi^b} &= \frac{A + \theta - \sqrt{B^2 + 4\theta A}}{B + 2ab(\theta - 2) + (\theta - 4)}, \\ \text{with } A &= 2(\theta - 2)(1 + ab) \text{ and } B = b(\theta - 4) + 2a(\theta - 2).\end{aligned}$$

Dispersion can be a stable equilibrium for both high and low trade costs. However, depending on the value of parameters, the economic existence of  $\underline{\phi^b}$  may vanish. If agglomeration forces are strong enough (i.e if  $\mu$  is large and  $\sigma$  is low), dispersion is stable only when trade is relatively free, as Figure 3 reveals<sup>10</sup> by plotting  $\Lambda(\phi)$  and  $\Gamma(\theta)$  with respect to trade costs and for different value of  $\mu$ .

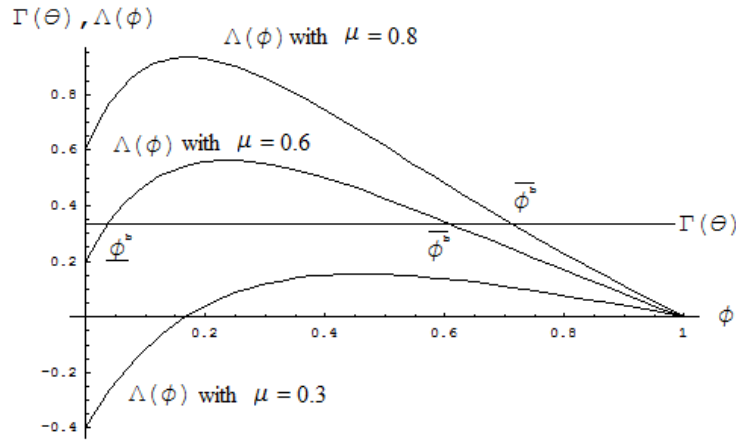


Figure 3

In a traditional economy, in which agricultural goods represent a major portion of consumption ( $\mu = 0.3$ ), regardless of the value of trade costs, dispersion is always stable (indeed  $\Gamma(\theta) > \Lambda(\phi)$ ). But if the share of income devoted to industrial good increases ( $\mu = 0.6$ ), dispersion is stable only for at low and high values of trade costs. Finally, in a modern economy, in which industrial goods represent the main consumption ( $\mu = 0.8$ ), dispersion is only stable when trade is liberalized i.e  $\phi \in [\overline{\phi^b}, 1]$  (as in Murata and Thisse [2005]).

In turn, dispersion is stable for high trade costs i.e  $\phi \in [0, \underline{\phi^b}]$  (as in Krugman [1991]) if and only if there are no urban costs (in that case  $\Gamma(\theta) = 0$ ) and the agglomeration forces are small enough (e.g in Figure 3 at  $\mu = 0.3$ ). Thus, Krugman [1991] imposes an assumption of "no black holes," which limits the agglomeration forces:  $\frac{\sigma-1}{\sigma} > \mu$ .

The parameters in Figure 3 satisfy a different assumption:

$$\mu > \frac{\sigma - 1}{2} \tag{23}$$

<sup>10</sup>Parameters:  $\sigma = 1.5, f = 1, L = 0.5, \theta = 0.8$

To understand why this condition is useful, consider the break points in terms of commuting costs.<sup>11</sup> This analysis is interesting because it can describe the interplay between commodities' transportation costs and workers' commuting costs in the formation of the space economy. From Proposition 1 we know that the symmetric equilibrium is stable if and only if  $\Lambda(\phi) < \frac{\theta/2}{2-\theta}$ , which is equivalent to:

$$\theta > \frac{4\Lambda(\phi)}{1+2\Lambda(\phi)} \equiv \theta^b. \quad (24)$$

Appendix B shows that  $\theta^b$  is discontinuous with respect to  $\phi$ , however if the no-black-hole condition (23) holds then  $\theta^b$  becomes continuous  $\forall \phi \in [0, 1]$ . This condition allows for the evacuation of discontinuities and thus situations in which neither agglomeration nor dispersion is stable.

### 3.2 When is the Core-Periphery pattern sustainable?

To determine whether the total agglomeration in region 1 is a stable equilibrium, this investigation must determine whether the relative real wage in region 1 is higher than  $1(\Omega|_{h_1=1} > 1)$ , with  $\Omega|_{h_1=1}$  given by:

$$\Omega|_{h_1=1} = \frac{(2-\theta)\phi^{1-a}}{1+\phi^2-b(1-\phi^2)}. \quad (25)$$

This expression allows to write:

**Proposition 2** *The Core-Periphery equilibrium is stable for intermediate value of trade costs ( $\phi \in [\underline{\phi}^s, \overline{\phi}^s]$ ) if and only if agglomeration forces and urban costs are low enough (i.e if  $a \in [0, 1[$  and  $\theta \in [0, 2 \left[ 1 - \frac{1-b}{1+a} \left( \frac{(1-a)(1-b)}{(1+a)(1+b)} \right)^{-\frac{1-a}{2}} \right]$ ). If agglomeration forces are strong enough  $a \in [1, \infty[$ , agglomeration is stable for a wide range of trade costs ( $\phi < \overline{\phi}_s$ ) whatever the level of urban costs. With  $\underline{\phi}^s$  and  $\overline{\phi}^s$  given implicitly by:*

$$\frac{(2-\theta)\phi^{1-a}}{1+\phi^2-b(1-\phi^2)} = 1 \quad (26)$$

**Proof.** From (19) we get:

$$\Omega|_{h_1=1} = \frac{(2-\theta)\phi^{1-a}}{1+\phi^2-b(1-\phi^2)}.$$

and since the Core-Periphery equilibrium is stable if and only if  $\Omega|_{h_1=1} > 1$ , we find the implicit condition (26). Now we need to analyse how many roots are obtained from  $\Omega|_{h_1=1} = 1$ . Firstly observe that  $\Omega|_{h_1=1}$  can be bell-shaped with respect to the freeness of trade, indeed a log differentiation of  $\Omega|_{h_1=1}$  gives:

$$\frac{d \Omega|_{h_1=1} / \Omega|_{h_1=1}}{d\phi/\phi} = \left[ (1-a) - \frac{\phi^2(1+b)}{1-(1-\phi^2)(1+b)/2} \right] / \Omega|_{h_1=1}$$

---

<sup>11</sup>See also Murata and Thisse [2005].

and thus if  $a < 1$  the first right-hand term into the bracket is positive and since the second term is increasing in  $\phi$  and equals to zero at  $\phi = 0$ , the derivative is clearly positive up to some critical value of  $\phi$  and negative after. Solving  $\frac{d\Omega|_{h_1=1}/\Omega|_{h_1=1}}{d\phi/\phi} = 0$ , the top of the bell is at  $\phi = \sqrt{\frac{(1-a)(1-b)}{(1+a)(1+b)}}$  denoted  $\phi^{s \max}$  hereafter. Thus if  $a < 1$ ,  $\Omega|_{h_1=1}$  is below the unity around extreme values of trade freeness ( $\Omega|_{h_1=1, \phi=1} = \frac{2-\theta}{2} < 1$  and  $\Omega|_{h_1=1, \phi=0} = 0$  only if  $a < 1$ ) and above it if  $\Omega|_{h_1=1, \phi=\phi^{s \max}} > 1$ . By inserting  $\phi^{s \max}$  in (25), the condition  $\Omega|_{h_1=1, \phi=\phi^{s \max}} > 1$  is verified if  $\theta < 2 - \frac{1+(\phi^{s \max})^2 - b(1-(\phi^{s \max})^2)}{(\phi^{s \max})^{1-a}} = 2 \left[ 1 - \frac{1-b}{1+a} \left( \frac{(1-a)(1-b)}{(1+a)(1+b)} \right)^{-\frac{1-a}{2}} \right]$ . To sum up for  $a \in [0, 1[$  (or equivalently for  $\mu \in [0, \sigma - 1[$  because  $a = \frac{\mu}{\sigma-1}$ ) and  $\forall \theta \in [0, 2 \left[ 1 - \frac{1-b}{1+a} \left( \frac{(1-a)(1-b)}{(1+a)(1+b)} \right)^{-\frac{1-a}{2}} \right]$ , the equation (26) has two solutions,  $\underline{\phi}_s$  and  $\overline{\phi}_s$ , between which the agglomerative equilibrium is sustainable ( $\phi \in [\underline{\phi}_s, \overline{\phi}_s]$ ). If  $a > 1$ ,  $\frac{d\Omega|_{h_1=1}/\Omega|_{h_1=1}}{d\phi/\phi} < 1$  hence  $\underline{\phi}_s$  does not exist and  $\overline{\phi}_s$  exists only if  $\Omega|_{h_1=1, \phi=0} > 1$  which is always verified because  $\lim_{\phi \rightarrow 0} \phi^{1-a} = +\infty$  if  $a > 1$  (or equivalently  $\mu > \sigma - 1$ ). ■

The intuition behind this proposition is straightforward. At high trade costs, agglomeration is not possible because (with low agglomeration forces) servicing a distant market is prohibitively expensive. Conversely, at very low trade costs, agglomeration is not really necessary, because trade is cheap and agglomeration is expensive in reason of urban costs. If urban costs are not too high, agglomeration will be stable for intermediate trade costs.

This Proposition 2 only gives an implicit expression of  $\phi$  that resolves  $\Omega|_{h_1=1} = 1$ , but it is easy to determine the expression of commuting costs that resolves it, indeed one obtain  $\theta = 2 - \frac{1+\phi^2 - b(1-\phi^2)}{\phi^{1-a}}$ . Then the Core-Periphery equilibrium is stable if  $\Omega|_{h_1=1} > 1$ , which is verified under the condition that:

$$\theta < 2 - \frac{1 + \phi^2 - b(1 - \phi^2)}{\phi^{1-a}} \equiv \theta^s. \quad (27)$$

It is possible now to compare  $\theta^s$  and  $\theta^b$ , in order to analyse multiple equilibria.

### 3.3 Trade costs versus commuting costs

Agglomeration and dispersion are simultaneously stable if  $\theta^b < \theta < \theta^s$ , then by using (24) and (27) the condition that allows three stable equilibria i.e  $\theta^b - \theta^s < 0$  is<sup>12</sup>:

$$I \equiv \frac{4\Lambda(\phi)}{1 + 2\Lambda(\phi)} - 2 + \frac{1 + \phi^2 - b(1 - \phi^2)}{\phi^{1-a}} < 0 \quad (28)$$

By analyzing this condition, the following result emerges:

<sup>12</sup>More than a decade after the seminal paper of Krugman [1991], Robert-Nicoud [2005] has proved that the sustain point occurs at a higher level of trade costs than the break point. Murata and Thisse [2005] lead the same analysis concerning commuting costs and shows that  $\theta^s > \theta^b$ .

**Proposition 3** *If  $\mu > \frac{\sigma-1}{2}$  then agglomeration and dispersion can be simultaneous stable (i.e.  $\theta^b < \theta^s$ ).*

**Proof.** We have to show that  $I = \theta^b - \theta^s < 0$ , as a look at Figure 4 indicates. The curve  $I$  has the same vertical asymptotes than  $\theta^b$ . Hence by assuming  $\mu > \frac{\sigma-1}{2}$  (see Appendix B<sup>13</sup>) and because  $I|_{\phi=1} = \frac{dI}{d\phi}|_{\phi=1} = 0$  while  $\frac{d^2I}{d\phi^2}|_{\phi=1} = 2(a+b)^2 > 0$  we can conclude that  $I$  is concave downward and always negative  $\forall \phi \in [0, 1]$ . Thus  $\theta^b < \theta^s$ . ■

In order to illustrate how these critical points vary according to trade costs Figure 4.a<sup>14</sup> plots  $\theta^s$  and  $\theta^b$  for different values of  $\mu$ .

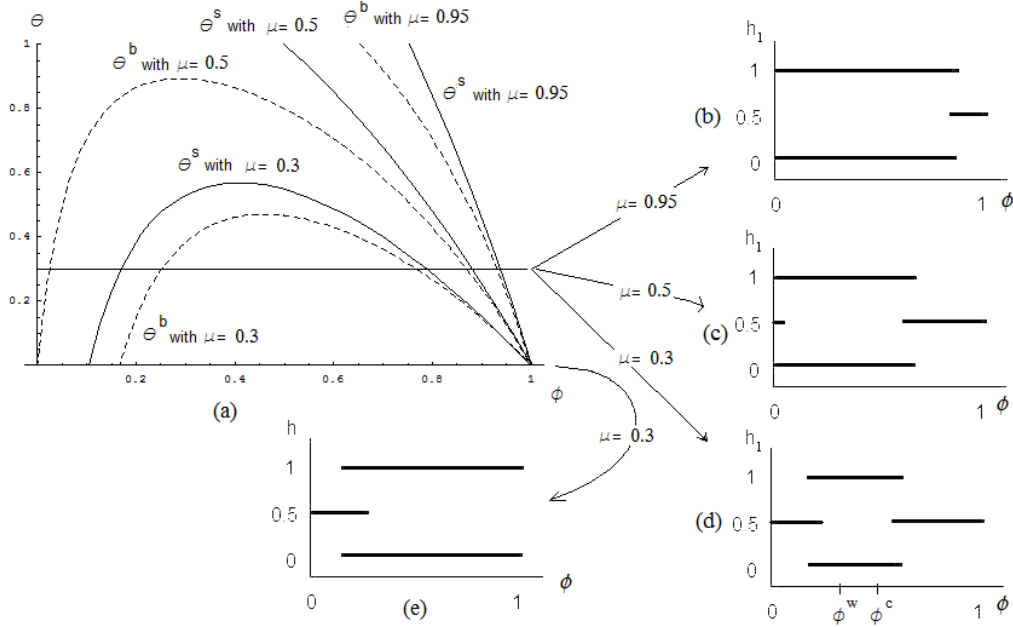


Figure 4

First observe what happens when the share of industrial goods is relatively high ( $\mu = 0.95$ ). In that case,  $\theta^s$  and  $\theta^b$  are strictly decreasing with respect to trade costs, and for a given level of urban cost (e.g.,  $\theta = 0.3$ ), the economy moves from agglomeration to dispersion when trade gets liberalized. This spatial configuration is schematically represented in Figure 4.b, in which the vertical axis indicates the share of skilled workers in

<sup>13</sup>In many simulations it has been checked what happens if  $\mu < \frac{\sigma-1}{2}$ . It has been found that (i) for high trade costs ( $\forall \phi \in [0, \frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}]$ ) we get  $\theta^b > 1$  (see Appendix B, Figure A) and  $\theta^s < 0$  which means that  $\forall \theta \in [0, 1]$  there is neither agglomeration nor dispersion in reason of discontinuities. Thus the condition  $\mu > \frac{\sigma-1}{2}$  really deserves its name of "no black hole condition"; (ii) for lower trade costs (i.e.  $\forall \phi \in [\frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}, 1]$ ) we get the same results than those presented at Figure 4 (this is also clearly illustrated by Figures 1 and 2 where the no black hole assumption has not been done but where trade freeness were higher than  $\frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}$ ).

<sup>14</sup>Parameters:  $\sigma = 1.5, f = 1, L = 0.5$



region 1, and the horizontal axis refers to the level of trade freeness. This figure represents, in the New Economic Geography jargon, a Tomahawk diagram.

Figure 4.a is more general than a standard Tomahawk diagram though, because every horizontal line reveals a different spatial configuration. For instance, without urban costs ( $\theta = 0$ ) but with low agglomeration forces ( $\mu = 0.3$ ), dispersion is stable for high trade costs, but agglomeration occurs when trade is relatively free (Figure 4.e).

If commuting costs are small enough (e.g.  $\theta = 0.3$ ) and agglomerative forces are not too strong (e.g.  $\mu = 0.3$ ), dispersion also can be stable for extreme values of trade costs and agglomeration at intermediate values. This spatial configuration appears in Figure 4.d, where the critical points  $\phi^w$  and  $\phi^c$  derive from Equations (20) and (21) are reported.<sup>15</sup> Between these two points, the agglomeration reward is quite strong, because the Core offers a higher nominal wage than does the Periphery—but also a lower cost of living.

## 4 Welfare

### 4.1 Agglomeration versus dispersion

Rather than investigating only skilled workers' relative welfare, this section offers a finer analysis that considers the individual welfare of the four interest groups, as given by:

$$\begin{aligned} V_1^h(h) &= c \frac{(1 - \theta h_1/2)w_1}{\Delta_1^{-a}}, & V_2^h(h) &= c \frac{(1 - \theta h_2/2)w_2}{(\Delta_2)^{-a}} \\ V_1^L(h) &= c \frac{1}{\Delta_1^{-a}}, & V_2^L(h) &= c \frac{1}{(\Delta_2)^{-a}} \end{aligned}$$

with  $c = \mu^\mu(1 - \mu)^{1-\mu}$ . In the rest of this subsection we drop the term  $c$ .

The objective therefore is to examine these four expressions for the two opposite equilibria, agglomeration and dispersion, and determine which offers the better social outcome. Skilled workers clearly prefer agglomeration to dispersion when trade costs are relatively high ( $\phi < \phi_h^p = \frac{4-3\theta}{4-\theta}$ ).

Indeed a skilled worker prefers agglomeration to dispersion when  $V_1^h(1) > V_1^h(\frac{1}{2})$ , and with

$$\begin{aligned} V_1^h(1) &= \frac{bL}{(1-b)} [2 - \theta]^a \\ V_1^h(\frac{1}{2}) &= \frac{2bL}{1-b} \left[ \frac{(4 - \theta)(1 + \phi)}{8} \right]^a \end{aligned}$$

such an inequality is verified when:

$$\phi < \phi_h^p = \frac{4 - 3\theta}{4 - \theta} \quad (29)$$

This result is in accordance with Charlot et al. [2006], who analyze welfare in a model without commuting costs and find that "whatever the level of transport costs, all

<sup>15</sup>Precisely we find  $\underline{\phi}^s = 0.17 < \underline{\phi}^b = 0.25 < \phi^w = 0.66 < \phi^c = 0.762 < \overline{\phi}^b = 0.767 < \overline{\phi}^s = 0.79$

skilled workers prefer agglomeration to dispersion." Here with  $\theta = 0$ , skilled workers prefer agglomeration from autarky to free trade, because in a such case  $\phi_h^p = 1$ .

Next, regarding the situation of immobile and unskilled workers, at the Core, they do not care about transaction costs. They have nothing to import, because all varieties are produced in region 1.

$$V_1^L(1) = \left[ \frac{2 - \theta}{2} \right]^a \quad (30)$$

However, even if they do not commute, commuting costs affect their welfare, because such costs reduce the entrepreneurial force in the manufacturing sector and raise the cost of living everywhere.

Notice that by assuming both that unskilled workers only enter the marginal cost and that a CRS good that can be freely exported, this investigations requires the marginal cost of production to be the same in the two regions, independent of the specialization pattern of the region. In turn, whatever the share of the IRS activity in each region, (1) the mill price of varieties is the same in the two regions, and (2) the nominal income of unskilled workers remains identical. Thus, these standard assumptions have crucial implications for not only the study of the agglomeration process but also welfare analyses. In particular, only differences in the cost of living matter for unskilled workers.

Under dispersion, unskilled workers' welfare are given by:

$$V_1^L\left(\frac{1}{2}\right) = V_2^L\left(\frac{1}{2}\right) = \left[ \frac{(4 - \theta)(1 + \phi)}{8} \right]^a \quad (31)$$

Then under dispersion trade liberalization is welfare-enhancing.<sup>16</sup> These two equations and previous results in turn indicate that:

**Proposition 4** *Whatever the value of commuting costs and trade costs there is no conflict of interest between skilled and unskilled workers within the same city : in region 1 agglomeration is unanimously preferred to dispersion if and only if trade costs are high enough ( $\phi < \frac{4-3\theta}{4-\theta} \equiv \phi^p$ ), otherwise dispersion is preferred by all.*

**Proof.** Concerning unskilled workers, those in region 1 prefer agglomeration to dispersion under the condition that  $V_1^L(1) > V_1^L(\frac{1}{2})$ . By using equation (30) and equation (31) such a result is obtained when:

$$\phi < \phi_L^p = \frac{4 - 3\theta}{4 - \theta} \quad (32)$$

and we have shown previously that skilled workers prefer agglomeration to dispersion when  $\phi < \phi_h^p = \frac{4-3\theta}{4-\theta}$ . Thus  $\phi_h^p = \phi_L^p$  and there is no conflict of interests : agglomeration is preferred to dispersion at the unanimity in region 1 when  $\phi < \frac{4-3\theta}{4-\theta} \equiv \phi^p$ . ■

This result reflects the finding that all forces that increase the desirability of concentration in the Core decrease and may even disappear when it comes to trade freedom. At the opposite, the land market crowding effect remains constant, so the price of agglomeration, in terms of commuting costs, becomes too high for all the inhabitants of the Core when trade gets liberalized. This result also might be explained by commuting costs, which

<sup>16</sup>This is true provided there is no change in the nature of the equilibrium.

generate waste in the Core by decreasing the labor force available to produce varieties, which means that love for variety is less satisfied under the agglomerative equilibrium.

Welfare of unskilled in region 2 when agglomeration occurs in region 1 is given by:

$$V_2^L(1) = \left[ \frac{2-\theta}{2} \phi \right]^a \quad (33)$$

Therefore, in region 2, trade costs have a real importance, and their decline enhances welfare. Hence,

**Proposition 5** (a) *Agglomeration is always detrimental to unskilled workers at the Periphery, and (b) dispersion is Pareto improving for all workers if and only if trade is free enough ( $\phi < \frac{4-3\theta}{4-\theta} \equiv \phi^p$ ).*

**Proof.** Unskilled workers in region 2 prefer dispersion to agglomeration if:

$$V_2^L(1/2) - V_2^L(1) > 0$$

and according to (30) and (31) this relation is verified when  $\phi < \phi_{L^*}^p = \frac{4-\theta}{4-3\theta}$ , which is always satisfied since  $\frac{4-\theta}{4-3\theta} > 1$ . Unskilled workers of region 2 always prefer dispersion to agglomeration. From this observation and previous proposition, it is clear that whereas unskilled workers in region 2 are always against agglomeration, workers in region 1 are in favor of such an equilibrium when the degree of trade liberalization is lower than  $\phi^p$ . Beyond this level though, dispersion enhances the welfare of everyone. ■

A similar result comes from Ottaviano and Thisse [2002] in a different setting without urban costs. As in their model by leaving one region (i.e., the dispersed situation), skilled workers generate a negative externality for the welfare of the remaining unskilled workers but a positive impact on the welfare of unskilled workers in the emerging Core (when some impediments to trade exists  $\phi < \phi^p$ , it is clear that  $V_L(1) > V_L(1/2) = V_L^*(1/2) > V_L^*(1)$ ).

## 4.2 Equilibrium versus optimum

This section considers whether firms' spatial distribution, as given by trade liberalization, is optimal. This analysis relies on the critical values of urban costs.

From proposition 5 we know that dispersion is Pareto improving if  $\phi < \frac{3\theta-4}{\theta-4}$ , then by rearranging this inequality we obtain that dispersion is improving for all if:

$$\theta > \frac{4(1-\phi)}{3-\phi} \equiv \theta^p$$

By comparing  $\theta^p$  and  $\theta^b$  one can find the following result:

**Proposition 6** *If the consumption of industrial goods is high enough ( $\mu > \frac{3(\sigma-1)}{2}$ ) then the market leads to an equilibrium of dispersion which is always optimal  $\forall \theta \in [\theta^p, \theta^b]$ .*

**Proof.** Dispersion is not optimal for  $\forall \theta \in [\theta^b, \theta^p]$ . This occurs in particular for high trade costs (see Figure 5.b) then a condition that avoids this situation is  $\theta^b \Big|_{\phi=0} > 1$ .

This condition is verified if  $\mu > \frac{3(\sigma-1)}{2}$ . One can also verify that under this condition we have  $\left. \frac{d\theta^b}{d\phi} \right|_{\phi=1} > \left. \frac{d\theta^p}{d\phi} \right|_{\phi=1}$ . Indeed  $\left. \frac{d\theta^b}{d\phi} \right|_{\phi=1} > \left. \frac{d\theta^p}{d\phi} \right|_{\phi=1}$  if  $\mu > \frac{\sigma(\sigma-1)}{2\sigma-1}$  and it is clear that  $\frac{3(\sigma-1)}{2} > \frac{\sigma(\sigma-1)}{2\sigma-1}$  for all  $\sigma > 1$ . Lastly observe that  $\theta^p = 1$  if  $\phi = 1/3$  and notice that  $\forall \mu \in [\frac{3(\sigma-1)}{2}, 1]$  we get  $\theta^b \Big|_{\phi=1/3} = \frac{2(1+a(b-2)-2b)}{-1+a(b-2)-b} > 1$ . Indeed  $\frac{2(1+a(b-2)-2b)}{-1+a(b-2)-b} > 1$  if  $\mu \in [-\frac{3+5\sigma-\sqrt{9-18\sigma+13\sigma^2}}{2}, 1]$  and  $\frac{3(\sigma-1)}{2} > -\frac{3+5\sigma-\sqrt{9-18\sigma+13\sigma^2}}{2}$  for all  $\sigma > 1$ . ■

This result indicates that agglomeration policies are not recommended from the Pareto criterion if  $\mu > \frac{3(\sigma-1)}{2}$  and  $\forall \theta \in [\theta^p, \theta^b]$ .

To illustrate this result and determine whether firms' spatial distribution, as given by trade liberalization, is optimal, Figure 5 compares  $\theta^s$ ,  $\theta^b$  and  $\theta^p$ . According to Figure 5.a, when the manufacturing sector is small relative to the agricultural sector ( $\mu = 0.3$ ), the market can reach an equilibrium of dispersion that is not optimal  $\forall \theta \in [\theta^b, \theta^p]$ . In particular, in this traditional economy, if trade costs are relatively high, dispersion is always detrimental to skilled workers and unskilled workers in the potential Core. However, because critical points of spatial configuration depend on  $\mu$  while  $\theta^p$  not, unanimity in favor of dispersion can be systematically attained in this market.

As Figure 5.b shows, in a modern economy for which the agricultural sector has lesser importance ( $\mu = 0.5$ ), the range of parameters over which dispersion occurs in a non-optimal fashion decreases. Figure 5.c ( $\mu = 0.99$ ) illustrates that dispersion, when it occurs in a modern economy, is always a Pareto improvement.

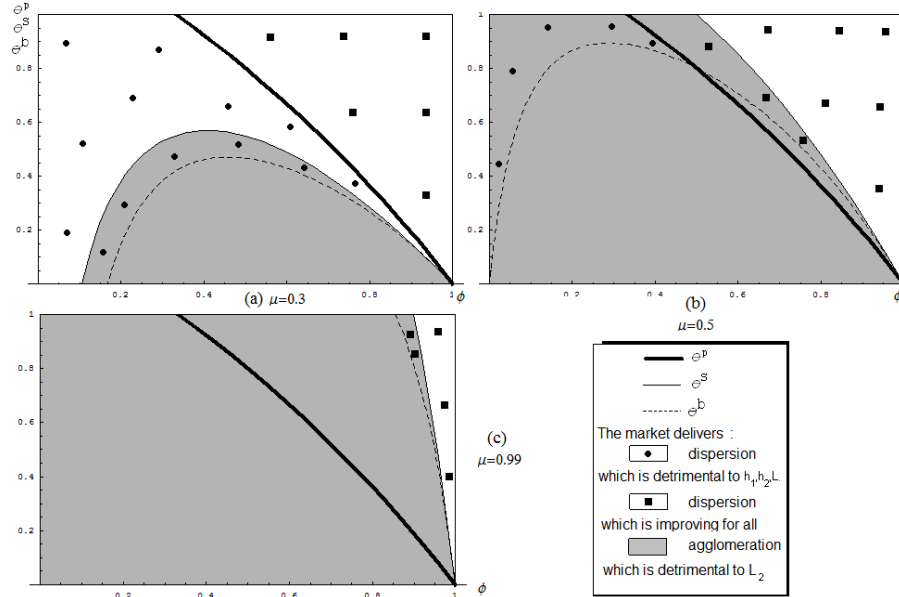


Figure 5

To conclude, this analysis leads to an interesting corollary to Propositions 3 and 7:

**Proposition 7** *If the consumption of industrial goods is high enough ( $\mu > \frac{3(\sigma-1)}{2}$ ) then*

for all levels of commuting costs that belong to  $[\theta^b, \theta^s]$  dispersion and agglomeration are stable but dispersion is better for all individuals (i.e.  $\theta^s > \theta^b > \theta^p$ )

**Proof.** Proposition 3 proves that if  $\mu > \frac{\sigma-1}{2}$  then  $\theta^s > \theta^b$  and Proposition 6 proves that if  $\mu > \frac{3(\sigma-1)}{2}$  then  $\theta^b > \theta^p$  thus if  $\mu > \frac{3(\sigma-1)}{2}$  one gets the following ranking:  $\theta^s > \theta^b > \theta^p$ .

■

Such a result is not common in existing literature. Robert-Nicoud [2006], Ottaviano and Robert-Nicoud [2006], and Charlot et al. [2006] all find that agglomeration and dispersion cannot be Pareto ranked and then carried out a cost-benefit analysis<sup>17</sup>; Sudekum [2008] and Pflüger and Sudekum [2008.b] use a specific social welfare function (utilitarian). Here the Pareto criterion is sufficient to obtain a result in which dispersion may be the best situation, assuming some specific conditions are met. In line with Gaigné [2006], this study reveals that "the analysis of the desirability of agglomeration merits more attention."

## 5 Conclusion

"The models turns Sartre's 'Hell is other people' on its head: agglomeration is unambiguously good for you. Because the cost of living is lower in the Core, it is always better to live there than in the periphery." Neary [2001]

The ambiguous desirability of the spatial concentration of activities is often emphasized by NEG models, because agglomeration entails prejudice against some, including immobile workers who live at the Periphery, while also making lives of others easier, such as the skilled workers or entrepreneurs who can find more outlets in the Core. However, as noted by Peter Neary, many models in this field suggest agglomeration as a process that benefits skilled workers or entrepreneurs, even though that standpoint is debatable. In Japan, agglomeration in cities like Tokyo, Osaka, and Nagoya (the three largest metropolitan areas) seems excessive to some entrepreneurs and skilled workers, who reject urban costs and started moving away during the 1970s.<sup>18</sup>

Because it is not entirely satisfying to use a model in which the cost of living is always lower in the Core, we introduce commuting costs and land rent into one of the workhorse models of the NEG. This introduction alters some conclusions about spatial configuration and welfare. In particular, Propositions 1, 2, and 3 list some conditions in which agglomeration and/or dispersion are stable. Proposition 4 shows that there is no conflict of interest between inhabitants of the big city who prefer agglomeration of activities when trade costs are relatively high. However, Proposition 5 indicates that this spatial configuration is always detrimental to Peripheral workers, whereas dispersion can be a Pareto-efficient outcome if trade costs are low enough. Proposition 7 suggests that in a modern economy, in which the consumption of industrial goods is quite high compared with the consumption of agricultural goods, decentralization policies that favor dispersion may be useful if agglomeration occurs. On the contrary, Proposition 6 indicates that in

<sup>17</sup>See Candau and Fleurbaey [2009] for a criticism on this method and a new proposition to compare welfares

<sup>18</sup>See Fujita et al. [2004], who argue that the predictions of the CP model can be verified only between 1955 and 1962.

some conditions, agglomeration policies are perhaps not appropriate, according to the Pareto criterion, if dispersion emerges spontaneously.

This study represents an initial first step in understanding how trade costs and urban costs may interact in the formation of cities. The spatial patterns adopted should be improved. In particular, this research focuses on monocentric cities, in which unskilled workers live and work in the suburbs and do not pay land rent. If such an assumption provides useful as a first approximation, it also deserves to be improved and extended in further research<sup>19</sup>.

Moreover, assuming perfect mobility for skilled workers makes sense at a regional level but not at the international scale. In recent periods, skilled workers have been slightly more mobile than unskilled workers, but this scenario has not always been the case (e.g., early migrations in Europe or the United States<sup>20</sup>). Furthermore, mobility remains very low for both skilled and unskilled workers. Thus these results require verification through other economic geography models that assume no or imperfect labor mobility.<sup>21</sup>

## • References

ALONSO-VILLAR O. (2008) A model of economic geography with demand-pull and congestion costs? *Papers in Regional Science* Volume 87 Issue 2, Pages 261 - 276.

BEHRENS K., ROBERT-NICOUD F. (2009). Krugman's Papers in Regional Science: the 100-dollar bill on the sidewalk is gone and the 2008 Nobel Prize well deserved. *Papers in Regional Science* 88(2), pp 467-489.

BRACKMAN S, GARRETSEN H (2009) Trade and Geography: Paul Krugman and the 2008 Nobel Prize for Economics. *Spatial Economic Analysis* 4: 1–19.

Brühlhart M. (2009) Is the new economic geography passé? *VoxEU* (7 January 2009)

CANDAU F. (2008a) - «Good Governance, Trade and Agglomeration », *Papers in Regional Science*. Vol 87 Issue 4, pp 483-504.

CANDAU F. (2008b). - « Urban Costs, Trade Costs and Tax Competition », *Revue d'Economie Politique*, Vol 118, 5, p. 625-661.

CANDAU F. (2008c). - « Entrepreneur' Location Choice and Public Policies, a Survey of the New Economic Geography », *Journal of Economic Surveys*. Vol. 22, Issue 5, pp. 909-952.

CANDAU F., FLEURBAEY M. (2009). - « Agglomeration and Welfare with Heterogeneous Preferences ». Available at SSRN: <http://ssrn.com/abstract=1393302>

CAVAILHES J., GAIGNE C. and THISSE J-F. (2004), Trade costs versus urban costs, *CEPR discussion paper* 4440.

COMBES, P-P, M LAFOURCADE, J-F THISSE and J-C TOUTAIN (2008) The Rise and Fall of Spatial Inequalities in France: A Long-Run Perspective. CEPR Discussion Paper 7017.

CHARLOT S., GAIGNE C., ROBERT-NICOUD F., THISSE J-F., (2006). - « Agglomeration and Welfare: the core-periphery model in the light of Bentham, Kaldor, and Rawls », *Journal of Public Economics* 90, p. 325-47.

---

<sup>19</sup>For example, the absence of interest conflicts between skilled and unskilled workers in the big city might be altered if the process of agglomeration affected the price of land for unskilled, immobile workers.

<sup>20</sup>See Hatton and Williamson [2008].

<sup>21</sup>Such as Krugman and Venables [1995], Tabuchi and Thisse [2002] or Murata [2003].

- DIXIT A.K., STIGLITZ J.E. (1977). - « Monopolistic competition and optimum product diversity », *American Economic Review* 67, p. 297-308.
- FORSLID R., OTTAVIANO G.I.P., (2003). - « An Analytically Solvable Core-Periphery Model », *Journal of Economic Geography* 3, 229-240.
- FUJITA M., HENDERSON J. V., KANEMOTO Y., MORI T., (2004). - « Spatial Distribution of Economic Activities in Japan and China ». In *Handbook of Urban and Regional Economics*. Volume 4 Eds J.V. Henderson, JF Thisse (Amsterdam, North-Holland).
- FUJITA M., KRUGMAN P., VENABLES A., (1999). - « The Spatial Economy ». Cambridge MA: MIT Press.
- FUJITA M., THISSE J-F (2009) New economic geography: An appraisal on the occasion of Paul Krugman's 2008 Nobel Prize in Economics. *Regional Science and Urban Economics* 39: 109-119
- GAIGNE C. (2006). - « The 'genome' of NEG models with vertical linkages. A comment on the welfare analysis » *Journal of Economic Geography* 6, 141-159.
- HATTON T., WILLAMSON J., (2008). - « Global Migration and the World Economy. Two Centuries of Policy and Performance ». Cambridge, MIT Press.
- HEAD K., MAYER T. (2004). - « The empirics of agglomeration and trade », in *Handbook of Regional and Urban Economics*, Volume 4 Eds J.V. Henderson, JF Thisse (Amsterdam, North-Holland).
- HELPMAN H. (1998). - « The Size of Region », In: D. Pines, E. Sadka, and I. Zilcha (eds.). *Topic in public economics. Theoretical and Applied Analysis*. Cambridge: Cambridge University Press, p. 33-54.
- KIM S (1998). - « Economic Integration and Convergence: US Regions, 1840-1987 », *Journal of Economic History*, 58(3): 659-683.
- KRUGMAN P (2008), Increasing returns, Nobel Prize lecture available at [http://nobelprize.org/nobel\\_prizes/economics/laureates/2008/](http://nobelprize.org/nobel_prizes/economics/laureates/2008/).
- KRUGMAN P. (1991). - « Increasing Returns and Economic Geography », *Journal of Political Economy* 99, p. 483-499.
- KRUGMAN P., LIVAS ELIZONDO R. (1996). - « Trade policy and the third world metropolis », *Journal of Development Economics* 49, p. 137-150.
- KRUGMAN P., VENABLES A. (1995). - « Globalization and the inequality of nations », *Quarterly Journal of Economics* 110, p. 857-880.
- MURATA Y., (2003). - « Product diversity, taste heterogeneity, and geographic distribution of economic activities : Market vs. non-market interactions », *Journal of Urban Economics* 53, p 126-144.
- MURATA Y., THISSE J-F., (2005). - « A simple model of economic geography à la Helpman-Tabuchi », *Journal of Urban Economics*, p. 137-155.
- NEARY P. (2001). - « Of hype and hyperbolas : introducing the new economic geography », *Journal of Economic Literature* 39, p. 536-561.
- OTTAVIANO G.I.P., THISSE JF., Integration, agglomeration and the political economics of factor mobility, *Journal of Public Economics* 83, 429-456.
- OTTAVIANO G.I.P., ROBERT-NICOUD F., (2006). - « The 'genome' of NEG models with vertical linkages: A positive and normative synthesis », *Journal of Economic Geography* 6, p 113-139.
- PFLUGER M., SUDEKUM J., (2008.a). - « A synthesis of footlose entrepreneur new economic geography models: when is agglomeration smooth and easily reversible? *Journal*

of *Economic Geography* 8, p 39-54.

PFLUGER M., SUDEKUM J., (2008.b). - « Integration, Agglomeration and Welfare », *Journal of Urban Economics* 63:2, p 544-566.

ROBERT-NICOUD F., (2005). - « The structure of simple New Economy Geography models (Or, on identical twins) », *Journal of Economic Geography* 5, p 201-34.

ROBERT-NICOUD F., (2006). - « Agglomeration and Trade with Input-Output Linkages and Capital Mobility », *Spatial Economic Analysis* 1, p 101-26.

SUDEKUM J., (2006). - « Agglomeration and Regional Costs-of-Living », *Journal of Regional Science* 46, p 529-543.

SUDEKUM J., (2008). - « Regional Costs-of-Living with Congestion and Amenity Differences - An Economic Geography Perspective », Forthcoming in *Annals of Regional Science*.

TABUCHI T. (1998). - « Urban agglomeration and dispersion: a synthesis of Alonso and Krugman », *Journal of Urban Economics* 44, p. 333-351.

TABUCHI T., THISSE J-F., (2002). - « Taste heterogeneity, labor mobility and economic geography », *Journal of Development Economics* 69, p. 155-177.

## A When is the symmetric equilibrium broken?

By letting for convenience

$$\Lambda_A = \frac{1 - \phi}{1 + \phi} \quad \text{and} \quad \Lambda_B = -\frac{1 - b - \phi(1 + b)}{1 - b + \phi(1 + b)} \quad (34)$$

we get:

$$\left. \frac{d\Omega}{dh_1} \right|_{h_1=1/2} = \left. \frac{d}{dh_1} \left( \frac{1 - \theta h_1/2}{1 - \theta h_2/2} \right) \right|_{h_1=1/2} + \left. \frac{d}{dh_1} \left( \frac{w_1}{w_2} \right) \right|_{h_1=1/2} + \left. \frac{d}{dh_1} \left( \frac{\Delta_1^a}{\Delta_2^a} \right) \right|_{h_1=1/2} \quad (35)$$

with:

$$\begin{aligned} \left. \frac{d}{dh_1} \left( \frac{1 - \theta h_1/2}{1 - \theta h_2/2} \right) \right|_{h_1=1/2} &= \frac{4\theta}{\theta - 4} \\ \left. \frac{d}{dh_1} \left( \frac{w_1}{w_2} \right) \right|_{h_1=1/2} &= 8 \frac{\theta - 2}{\theta - 4} \Lambda_A \Lambda_B \\ \left. \frac{d}{dh_1} \left( \frac{\Delta_1^a}{\Delta_2^a} \right) \right|_{h_1=1/2} &= 8a \frac{\theta - 2}{\theta - 4} \Lambda_A \end{aligned}$$

Thus from (35) we find that  $\left. \frac{\partial \Omega}{\partial h_1} \right|_{h_1=1/2} < 0$  if and only if  $\Lambda_A(a + \Lambda_B) < \frac{\theta/2}{2-\theta}$  which gives the proposition 1 with  $\Lambda(\phi) \equiv \Lambda_A(a + \Lambda_B)$  and  $\Gamma(\theta) \equiv \frac{\theta/2}{2-\theta}$

## B The "no black hole condition" revisited

Let's observe how  $\theta^b$  behaves with respect to  $\phi$ . This critical point has two vertical asymptote at  $\phi = \frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}$  and  $\phi = \frac{3+2ab+\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}$ . So we are going



to analyse  $\theta^b$  successively for  $\phi \in (-\infty, \frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}]$ ,  $\phi \in [\frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}, \frac{3+2ab+\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}]$  and  $\phi \in [\frac{3+2ab+\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}, +\infty)$ .

According to standard algebra  $\theta^b$  is concave upward for  $\phi \in (-\infty, \frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}]$  and attains its minimum at  $\phi = \frac{a(b^2-1)-2\sqrt{(1+ab)(1-b^2)}}{(1+b)(2+a+ab)}$  denoted  $\theta^{b \min}$  (not reported here for convenience but available on request). Figure A plots  $\theta^{b \min}$  with respect to  $\sigma$  for different values of  $\mu$  and shows that  $\theta^{b \min}$  is always higher than one. But by definition  $\theta < 1$  this means that  $\forall \phi < \frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}$  we always get  $\theta < \theta^b$  and thus dispersion is never stable.

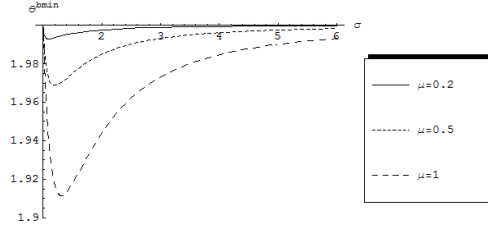


Figure A

But this situation can be avoided, indeed by definition  $\phi \in [0, 1]$  thus if the vertical asymptote  $\phi = \frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}$  is equal to zero, then we can forget what happens for  $\phi \in (-\infty, \frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}]$ . Hence we find that  $\frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab} < 0$  if  $\mu > \frac{\sigma-1}{2}$ .

Standard algebra also shows that  $\theta^b$  is concave downward for  $\phi \in [\frac{3+2ab-\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}, \frac{3+2ab+\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}]$  and cuts the horizontal axis at  $\phi = \frac{(a-1)(b-1)}{(1+a)(1+b)}$  and  $\phi = 1$ . This means that the second asymptote  $\phi = \frac{3+2ab+\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}$  is always higher than one. Then since by definition  $\phi \leq 1$  we don't need to know what happens on the last interval of existence (i.e.  $\phi \in [\frac{3+2ab+\sqrt{8+4a^2+12ab+b^2}}{1+2a+b+2ab}, +\infty)$ ). To conclude if  $\mu > \frac{\sigma-1}{2}$  then  $\theta^b$  can be bell-shaped  $\forall \phi \in [0, 1]$  and reaches a maximum (that can be smaller than one) denoted  $\theta^{b \max}$  at  $\phi = \frac{a(b^2-1)+2\sqrt{(1+ab)(1-b^2)}}{(1+b)(2+a+ab)}$ .