DETECTION OF HIGH AND LOW STATES IN STOCK MARKET RETURNS WITH MCMC METHOD IN A MARKOV SWITCHING MODEL

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Abstract

To detect abnormal states in stock market returns, this study considers seven indices, over an 21-year period, the Dow Jones, S&P500, Nasdaq, Nikkei225, the FTSE100, DAX, and CAC40. Three states are possible, namely a state of high rate of return, a state of low rate of return, both with high volatility and an intermediate state with low volatility. To determine the state of the market at each date, we study the returns using Markov Chain Monte Carlo method (Metropolis-Hastings algorithm). Then at a second time, using a Cramer's coefficient, we deduce association coefficients or "correlations" among the different states of the major stock exchange markets around the world. First, the associations were globally stronger during the subprime crisis than during the dot-com bubble period. Second, among European markets the Cramer's V is higher regardless of the period. Third, the associations between the Nikkei and the other markets indices are systematically lower, indicating the relative disconnection of the Japanese market.

1 Introduction

The availability of high frequency data for financial series and the development of mathematical and computer tools in recent decades support renewed research into the behavior of financial asset prices. The study of the statistical properties of asset returns, which are essential for managing portfolios of assets, is a vital component of the domain [1]. Several research areas thus have been explored including the problems of measuring volatility/risk assets, the phenomena of bubbles and financial crisis, the measures of abnormal returns, correlations or dependencies between assets and/or markets and contagion effects.

A particularly important question in financial analysis is how the volatility and the trend of stock prices behave. Such forecasts are essential to determinate options prices, portfolio optimization among others.

Some authors mainly focused on the mean return, we call also drift among a period. Since publications by Brown and Warner [2], [3], many articles sought to the measure security price performance, and particularly assess abnormal returns. Generally, the return of a security/stock is abnormal if it differs from a "normal" return, that is, from a particular benchmark. This benchmark may be a simple mean return, a return deduced from a CAPM model or a multi-factor model (e.g. the three-factor Fama and French model [17], [25]). However, these approaches face limitations, in that they require a risk-free return to estimate the normal return. In reality though, situations such as the actual debt crisis in Europe complicate the choice of a risk-free return. In addition, these methods essentially determine the abnormal returns of a security/stock. It also may be interesting to estimate abnormal returns of a market. We therefore propose another method to estimate abnormal returns that can be applied to both the return on a security and at the return on a market portfolio.

Many other authors focused on the behavior of the volatility. The first models are

based on autoregressive conditional heteroscedastic processes (ARCH), which should provide better measures of the dispersion and volatility of assets prices than a simple standard deviation [10], [1]. Some authors also are interested in modeling the tails of the distribution of returns (extreme values), such that they isolate the effects of shocks to the natural tendency of markets [8]. By extension, other studies focus on correlations/dependencies among stock returns, seeking possible crossmarket spillover effects in financial markets. Such work has taken several forms, from the analysis of contagion effects [20], [19], to the study of copulas [11].
However a simple observation shows that there are sequences of high and low volatilities. To take into account these facts, some authors introduce markov-switching parameters to ARCH models [15], [4], while Dueker [9] extended their approach to GARCH models to describe two stylized facts: conditional volatility can increase substantially (jumps) in a short time; "the stock-market volatility does not remain persistently two to three times above its normal level the same way it can persist at 30-40 percent above normal" ([9] p. 26). The standard approach considers that the parameters (one or several) in conditional volatility are subject to markov-switching, which permits to distinguish several states. Most of works retain three states, even if 4-states or 5-states models can be estimated. In [18] (p. 442),using a 4-states model, the authors conclude that the last state represents "only one observation that pertains to this regime ... the 1987 October crash of the stock market". In the standard case of three states, we generally distinguish: a state of high volatility, a state of low volatility and a medium state. If the first works use Maximum-Likelihood (ML) estimation, [18] adopt a MCMC method to estimate a time series model with autoregressive conditional heteroscedasticity and changes in regime.

The present study advances in the line of [13] [14], with the objective of revealing "abnormal" situations. Our main issue in modeling is how we define an "abnormal" return and according to which features we classify them. We highlight distinct regimes in the stock market returns based on the volatility but also on the drift.

Consequently, our research differs from the previous works as we seek to reveal switching not only in stock returns volatility but in drifts and volatility together. We focus on detecting the abnormal returns following [22] and [13]. We assume three states are possible. State 1 and 3 are considered to reflect periods of high volatility and respectively high and low drift while state 2 refers to periods with standard drift and low volatility. For instance the market is in state 1 during a bubble period and similarly in state 3 during a crisis period. However the controversy is not true. In a risk-on risk off approach, the transition itself to level 1 or 3 from the intermediate level is useful information since it characterizes a period of high uncertainty.

Meanwhile, following work by Black, Scholes and Merton in the 1970s, widespread uses of diffusion models for pricing options have been observed [12]. The use of variants of these models, assuming that the asset price is driven by an Ito process, involving a Brownian part, is also generalized for the study of returns.

In our study we consider that the return follows a diffusion process with a drift and a

volatility (covariance in multidimensional case) parameters. Moreover given the state of the market we assume those parameters are constants. Obviously they change from one state to another. Following this purpose we present a Bayesian approach based on a MCMC method/algorithm to estimate the parameters of the Markov switching model in each case together with the estimation of the state for each period.

We make the general assumption that the current price contains all the information such that the future price depends on the past only through the current observation. This assumption is necessary in finance for computing, for example, the option price. The other assumption we make pertains to the independence of returns, which is required to estimate the parameters of the model applying the Central Limit Theorem. Thus we model prices using a classical Black & Scholes diffusion for each state, an approach that is very convenient, because we can identify for each state its deterministic drift part and its random component, according to the volatility of the returns. Moreover the changing states from one date to another represent a transition matrix. The estimation relies on a Bayesian approach. In turn with this method, we can assess the value of the parameters while there is still liberty in them through observations made on the market.

We study the stock market returns of several developed countries. To do so, we being by detailing our model in Section 2, followed by MCMC method and the algorithms used in Section 3. Section 4 presents an application to weekly data for the Dow Jones, Nasdaq, S&P 500, Nikkei, FTSE, CAC 40, and Dax market indexes, over the period 1991-2011. Section 5 concludes.

2 The market model

We assume d assets that generate d risk sources. Let $T > 0$ be a time-finite horizon. We consider the d-dimensional Brownian motion $(W_t)_{t\in[0,T]}$ and $(\Omega, \mathcal{F}, \mathbb{P})$ the filtered probability space at stake, with Ω the set of events, $\mathcal{F} = (\mathcal{F}_t)_{t\in[0,T]}$, the natural Brownian filtration defined by $\mathcal{F}_t = \sigma(W_s; s \leqslant t)$. $\mathcal{F}_t$ contains all the information of the market up to date t. Originally the Black and Scholes model for the asset prices process denoted $(S_t)_{t\in[0,T]}$, with drift $b \in \mathbb{R}$, standard deviation $\sigma > 0$ and $d = 1$ has been defined by

$$dS_t = S_t(b + \frac{\sigma^2}{2})dt + S_t\sigma dW_t, \tag{1}$$

with $S_0 > 0$. Then for a given $s \in [0, T]$, the return process between dates s and t, denoted $(r(s,t))_{t\in[s,T]}$, and defined by $r(s,t) = \ln(\frac{S_t}{S_s})$ satisfies

$$dr(s,t) = bdt + \sigma dW_t, \tag{2}$$

with $r(s,s) = 0$. Since b and σ are fixed constants we have, for $t \geqslant s$,

$$r(s,t) = b(t-s) + \sigma(W_t - W_s). \tag{3}$$

One may notice that $W_t - W_s$ follows the normal distribution with mean 0 and variance $t-s$. Moreover it is independent from the sigma algebra $\mathcal{F}_s$, that is all the information contained in the market up to the date s. Now we present our model which is a variant from this model. This variant is multidimensional and b and σ depend on the time. Moreover the couple (b_t, σ_t) takes its values in a set that contains three elements.

The model

We denote $(S_t)_{t\in[0,T]} \in (\mathbb{R}_+^*)^d$ as the d-dimensional, strictly positive, underlying assets process. We consider that S has the following form,

$$\forall i \in \{1, \cdot, d\} \quad dS_t^i = S_t^i(b_t^i + \frac{1}{2}\sum_{j=1}^{d}(\sigma_t^{ij})^2)dt + S_t^i \sum_{j=1}^{d} \sigma_t^{ij} dW_t^j, \tag{4}$$

with $b_t \in \mathbb{R}^d, \sigma_t \in \mathbb{R}^{d\times d}$ $\mathcal{F}_t$-measurable. We define the log return as :

$$\forall 0 \leq s < t \leq T, i \in \{1, \cdot, d\}, \quad r^i(s,t) = \ln\left(\frac{S_t^i}{S_s^i}\right) \in \mathbb{R} \tag{5}$$

For a given $s \geqslant 0$, the Ito formula implies,

$$\forall t \geqslant s, \quad dr^i(s,t) = b_t^i dt + \sum_{j=1}^{d} \sigma_t^{ij} dW_t^j \tag{6}$$

We notice that for all $t \leqslant t' < t''$, then $r(t,t'') = r(t,t') + r(t',t'')$. Now we specify the form of the process $(b_t, \sigma_t)_{t\in[0,T]}$. We assume this couple can take three different constant values that is $(b_t, \sigma_t) \in \{(b_1,\sigma_1), (b_2,\sigma_2), (b_3,\sigma_3)\}$. We denote the discrete random Markov process $(Y_t)_{t\in[0,T]} \in \{1,2,3\}$ that specifies the state of the market. We define

$$b_t = b_{Y_t}, \quad \sigma_t = \sigma_{Yt}. \tag{7}$$

In that case (4) and (5) are well defined and have a unique solution.

Our purpose is to distinguish two states with low information inflow and low or high returns, labeled 1 and 3, and an intermediate regime, the second one, with lower variance and standard return. Following this idea we assume : $\forall j \in \{1, \cdots, d\}, b_1^j < b_2^j < b_3^j$ and for the matrix norm $\|.\|$ we have, $\|\sigma_1\|, \|\sigma_3\| > \|\sigma_2\|$.

We define $(Y_t)_{t\in[0,T]}$ as a piecewise constant process that can jump only on the time grid $\{t_0, \cdots, t_{n-1}\}$ with $t_k = \frac{kT}{N}$. We have then $Y_t = Y_{t_k}$ for all $t \in [t_k, t_{k+1}[$. In that

case we denote by $\mathcal{P} \in]0,1[^{3\times 3}$, its transition matrix. $\mathcal{P}$ gives the probability of being in a state at the next date, conditional on the current state, that is,

$$\forall k \in \{1, ..., n-1\}, \quad \mathbb{P}(Y_{t_{k+1}} = j | Y_{t_k} = i) = \mathcal{P}^{ij}. \tag{8}$$

We notice that the diagonal of $\mathcal{P}$ gives the probability to stay in the current state. We denote $\omega_t \in]0,1[^3$ the law of the random variable Y_t such that $\mathbb{P}(Y_t = i) = \omega_t^i$. We have then

$$\forall k \in \{1, ..., n\}, \quad \omega_{t_k} = \omega_0 \mathcal{P}^k \tag{9}$$

Its stable law is given by the solution of $\omega = \omega\mathcal{P}$. This law gives the probability of each state once equilibrium is reached. In such case $\omega^1 + \omega^3$ is the probability that the market is in a period of high volatility while ω^2 is the one of being in the intermediate regime. Heuristically we want ω^2 to be the highest probability.

The return process

Since $(Y_t)_{t\in[0,T]}$ is a piecewise constant process, then b_t and σ_t are constant for $t \in [t_k, t_{k+1}[$ and we can write the exact decomposition of the diffusion of the return process on time grid $\{t_0, \cdots t_N\}$ as :

$$r(t_{k+1}, t_k) \quad = \quad b^i_{Y_{t_k}}(t_{k+1} - t_k) + \sum_{j=1}^{d} \sigma^{ij}_{Y_{t_k}}(W^j_{t_{k+1}} - W^j_{t_k}). \tag{10}$$

Summing over k, we obtain,

$$\begin{aligned} &\forall l_1 < l_2 \in \{1, \cdots, n\}, \\ &r^i(t_{l_1}, t_{l_2}) = \sum_{k=l_1}^{l_2-1} b^i_{Y_{t_k}}(t_{k+1} - t_k) + \sum_{k=l_1}^{l_2-1}\sum_{j=1}^{d} \sigma^{ij}_{Y_{t_k}}(W^j_{t_{k+1}} - W^j_{t_k}). \end{aligned} \tag{11}$$

We notice that the Brownian increments $(W^j_{t_{k+1}} - W^j_{t_k})_{(k,j)\in\{0,\cdot,n-1\}\times\{1,\cdot,d\}}$ form a sequence of independent and identically distributed random variable following a centered normal distribution with variance $\frac{T}{n}$.

Remark 2.1. *From the independence of Brownian increments and properties on $(Y_t)_{t\in[0,T]}$ we can deduce that S, b_Y, and σ_Y are Markov processes. We also notice that for all s, t, s', t' on time grid $\{t_0, \cdots, t_n\}$, with $s < t \leqslant s' < t'$, then $r(s,t)$ is independent from $r(s',t')$.*

3 Estimation of the parameters

3.1 Bayesian approach

We define by $\theta = \big((b_1, \sigma_1), (b_2, \sigma_2), (b_3, \sigma_3), \mathcal{P}, Y_0, (Y_{t_k})_{k\in\{0,\cdot,n-1\}}\big)$ the set of parameters to estimate regarding the observations. We denote by $\mathcal{S}_n$ the set of observations until

t_n. To do so, we use a Bayesian approach. This method consists of establishing a distribution prior law, denoted ν, to θ. We then compute the most likely law, or posterior law, conditional to the observations : $\nu_{|\mathcal{S}_n} = \mathbb{P}(\theta|\mathcal{S}_n)$. Thus, we grant a liberty to the values to be selected for the parameter, though we still guide the calibration with respect to the law, ν, we choose for θ, and the parameters we apply to this law. Once we know the posterior law we estimate θ applying a Monte Carlo algorithm and the estimator is given by

$$\widehat{\theta} = \frac{1}{N}\sum_{i=1}^{N}\theta_i \tag{12}$$

with N the number of Monte Carlo steps and $(\theta_i)_{i\in\{1,\cdot,N\}}$ a sequence of independent and identically distributed random variables with law $\nu_{|\mathcal{S}_n}$.
We expose the prior distribution chosen for the different processes followed by the parameters of the equation (11). First of all we assume that b_i and σ_i are independent random variable. We rely on the following notations:

$$\begin{cases} b_i \sim \mathcal{N}(\mu_i, \Sigma_i) \\ (\sigma_i\sigma_i^*)^{-1} \sim W_p(\alpha_i) \\ (\mathcal{P})_i = (\mathcal{P}^{i1}, \mathcal{P}^{i2}, \mathcal{P}^{i3}) \sim Dir(\zeta_{i,1}, \zeta_{i,2}, \zeta_{i,3}) \\ Y_0 \sim \mathcal{U}\{1,2,3\} \\ \forall k \in \{0, \cdots, n-1\}, i \in \{1,2,3\}, \mathbb{P}(Y_{t_{k+1}} = i | Y_{t_k}, \mathcal{P}) = \mathcal{P}^{Y_{t_k} i} \end{cases} \tag{13}$$

where Dir refers to a Dirichlet distribution with three liberty degrees; $W_p(\alpha_i)$ is the Wishart distribution with mean α_i and p degrees of freedom, and $\mathcal{U}$ refers to the discrete uniform distribution.

Choice of the parameters for the prior law

The parameters μ, α, and ζ will determine the likely range of values that can take θ. For example, higher $|\mu_1|$ and $|\mu_3|$ imply a lower probability of their state. Indeed,

$$\tilde{\mathcal{P}}^{ij} = \mathbb{E}[\mathcal{P}^{ij}] = \frac{\zeta_{i,j}}{\sum_{k=1}^{3}\zeta_{i,k}}. \tag{14}$$

We denote $\tilde{\omega}$ the stable law of $\tilde{\mathcal{P}}$. We have,

$$\begin{aligned} \mathbb{E}[b_{t_k}] =& \mathbb{E}\big[\mathbb{E}[b_{Y_{t_k}}|\omega_{t_k}]\big] \\ =& \sum_{i=1}^{3}\mathbb{E}\big[\omega_{t_k}^i\mathbb{E}[b_i]\big] \\ =& \sum_{i=1}^{3}\mu_i\mathbb{E}[\omega_{t_k}^i] = \sum_{i=1}^{3}\mu_i\tilde{\omega}_{t_k}^i. \end{aligned} \tag{15}$$

The same way we have $\mathbb{E}[\sigma_{t_k}\sigma^*_{t_k}] = \sum_{i=1}^{3} \alpha_i^{-1}\tilde{\omega}^i_{t_k}$. Let us denote $\overline{b}$ and $\overline{\sigma}$ the Maximum Likelihood estimators of b and σ. A reasonable choice of parameters could be to impose μ, α and ζ such that

$$\begin{cases} \sum_{i=1}^{3} \mu_i \tilde{\omega}^i_{\cdot} = \overline{b}, \\ \sum_{i=1}^{3} \alpha_i^{-1}\tilde{\omega}^i = \overline{\sigma\sigma}^*. \end{cases} \tag{16}$$

Once we have selected these parameters, it remains to simulate the posterior law that is the prior law given the observations, and then to calculate the estimator using the formula 12. To achieve, we use a Markov Chain Monte Carlo method, and more precisely the Metropolis-Hasting algorithm.

3.2 Markov Switching and MCMC methodology

We recall we assume that the global asset process can switch states with respect to the evolution of the Markov chain $(Y_t)_{t\in[0,T]}$.

Heuristically, we try to find the best realization of this prior law that maximizes the likelihood of occurrence of the observations, and we estimate the parameters with Monte Carlo method. To obtain the posterior law, we use Metropolis-Hasting algorithm.
We do not go further in describing theoretical parametrization steps but rather refer the reader to [13]. Our purpose here is to identify different states in miscellaneous markets and their underlying common features. For example, we observe the Cramer's V between states identified in different markets around various periods. For this study we provide the Metropolis-Hasting algorithm applied to estimate the parameters of equation (11).

3.3 The Metropolis-Hasting algorithm

We do not describe this algorithm in detail because it has been well developed in prior literature. We recall simply that the retained approach relies on the use of a multiple-block Metropolis-Hastings algorithm , such that "the parameters are grouped into several distinct blocks, and then each block of parameters is updated in sequence by a Metropolis-Hastings step, conditioned on the most current value of the parameters in the remaining blocks" [6]. In other words, we have $\theta = (\theta_1, \cdots, \theta_l)$ and the posterior law of θ_i is the prior law of θ_i knowing $\mathcal{S}_n$ but also $\theta\backslash\{\theta_i\}$.

The idea of the algorithm remains the same as when $l = 1$, and relies on the Bayes formula. The methodology consists in defining a Markovian transition function f, a sequence of independent random variables $(Z^m)_{m\in\mathbb{N}}$ with density probability law $x \mapsto f(\theta^m, x)$, and a sequence of acceptance random variables $(\alpha^m)_{m\in\mathbb{N}} \in \{0, 1\}$ in order to define a Markov chain $(\theta^m)_{m\in\mathbb{N}}$ as follow

$$\theta^{m+1} = \alpha^m Z^m + (1 - \alpha^m)\theta^m. \tag{17}$$

It means that $\theta^{m+1} = Z^m \sim f(\theta^m, .)$ with probability $\mathbb{P}(\alpha^m = 1)$ and $\theta^{m+1} = \theta^m$ with probability $\mathbb{P}(\alpha^m = 0)$. If we choose

$$\mathbb{P}(\alpha^m = 1) = \frac{\mathbb{P}(Z^m|\mathcal{S}_n)f(Z^m, \theta^m)}{\mathbb{P}(\theta^m|\mathcal{S}_n)f(\theta^m, Z^m)} \tag{18}$$

it implies that θ^m, with law $\nu^m_{|\mathcal{S}_n}$, converges to a random variable with law $\nu_{|\mathcal{S}_n}$. We then compute the estimator with the formula (12) with $\theta_i = \theta^{m+i}$ when m is large enough to consider that we have reached the convergence field. The acceptance probability $\mathbb{P}(\alpha^m = 1)$ is calculated using the Bayes formula. Indeed

$$\mathbb{P}(\theta|\mathcal{S}_n) = \frac{\mathbb{P}(\theta)\mathbb{P}(\mathcal{S}_n|\theta)}{\mathbb{P}(\mathcal{S}_n)}. \tag{19}$$

Hence,

$$\mathbb{P}(\alpha^m = 1) = \frac{\mathbb{P}(Z^m)\mathbb{P}(\mathcal{S}_n|Z^m)f(Z^m, \theta^m)}{\mathbb{P}(\theta^m)\mathbb{P}(\mathcal{S}_n|\theta^m)f(\theta^m, Z^m)} \tag{20}$$

We give now the algorithm in our framework. First of all we give some necessary properties to ensure convergence in the Metropolis Hasting algorithm.

Definition 3.1. *Let $(M^m)_{m\in\mathbb{N}}$ be a Markov chain. Then,*

1. *M is reversible if : $\forall x, y \in \mathbb{R}^n, m \in \mathbb{N}$,*

$$\mathbb{P}(M^{m+1} = y|M^m = x)\mathbb{P}(M^m = x) = \mathbb{P}(M^{m+1} = x|M^m = y)\mathbb{P}(M^m = y). \tag{21}$$

2. *M is irreducible if : $\exists m_0 \in \mathbb{N}, m_0 > m$,*

$$\mathbb{P}(M^{m+m_0} = y|M^m = x) > 0. \tag{22}$$

3. *M is aperiodic if : $\forall x, y \in \mathbb{R}^n, m \geq \mathbb{N}$,*

$$\gcd\{m_0 \in \mathbb{N}^*, \mathbb{P}(M^{m+m_0} = y|M^m = x) > 0\} = 1. \tag{23}$$

Irreducibility means we can reach all states of the Markov chain with strictly positive probability. Aperiodicity signifies we cannot return to a state with regular time. It thereby avoids seasonality phenomena.

Now we expose the Metropolis-Hasting algorithm by block which is used in our study.

Proposition 3.1 (Metropolis-Hasting algorithm)**.** *Let $\mathcal{S}_n$ denotes the sample of n observations, N_0 be the number of steps before we run Monte Carlo, N be the number of Monte Carlo observations, and a prior law given by $\nu = (\nu_1, \cdots, \nu_l)$. For $x, y \in \mathbb{R}^d$, $j = 1, ..., l$, we introduce $f_j : (\mathbb{R}^d \times \mathbb{R}^d) \to \mathbb{R}_+$, s.a. $\int_{\mathbb{R}^n} f_j(x, y)dy = 1$, which constitutes the Markovian transition probability density function candidate for the parameters j, knowing $\theta \backslash \{\theta_j\}$.*

Algorithm 1 Bayes-Metropolis-Hasting

$m = 0$
Initialize θ
while $m \leq N + N_0$ **do**
  for $j = 1, ..., l$ **do**
    $\theta'_j \leftarrow \mathcal{X} \sim f_j(\theta_j, .)$
    $\alpha = \frac{\mathbb{P}(\theta'_j)\mathbb{P}(\mathcal{S}_n|\theta'_j) f_j(\theta_j, \theta'_j)}{\mathbb{P}(\theta_j)\mathbb{P}(\mathcal{S}_n|\theta_j) f_j(\theta'_j, \theta_j)}$
    $U \sim \mathcal{U}[0, 1]$
    if $U < \alpha$ **then**
      $\theta_j \leftarrow \theta'_j$
    end if
    if $m > N_0$ **then**
      $\hat{\theta}_j = \hat{\theta}_j + \theta_j / N$
    end if
  end for
  $m = m + 1$
end while
return $\{\hat{\theta}_1, ..., \hat{\theta}_l\}$ the set of estimated parameters

where $\mathbb{P}(\theta'_j) = \nu_j(\theta'_j)$ *and* $\mathbb{P}(\theta_j) = \nu_j(\theta_j)$.

Then, the Markov chain θ *thus defined is reversible, irreducible, and aperiodic, and it converges to a random variable with law* $\nu_{|\mathcal{S}_n}$. *Hence,* $\{\hat{\theta}_1, ..., \hat{\theta}_l\}$ *is the Bayesian estimator of the parameters of (11) with prior law* ν *and observations* $\mathcal{S}_n$.

Remark 3.1. *1. Since the state process takes discrete values, we adapt its Monte Carlo estimator at time* t_i *and define:*

$$\hat{Y}_{t_i} = \underset{j \in \{1,2,3\}}{argmax} \{y_j, y_j = Card\{(Y_{t_i})_{m > N_0}\}\}$$

2. We estimate each block, one at a time. Furthermore, the law of a block can depend on the value of another block. The proof of the convergence for this case is given in [5].

3. If the transition density function candidate is symmetric, then

$$\alpha = \frac{\mathbb{P}(\theta'_j)\mathbb{P}(\mathcal{S}_n|\theta'_j)}{\mathbb{P}(\theta_j)\mathbb{P}(\mathcal{S}_n|\theta_j)},$$

which offers a likelihood fraction. Heuristically, we choose the more likely realization under the law ν_j.

3.4 Choice of the transition density functions

We show some possible choices for the transition density candidate similar to random walks or more generally to symmetric density functions. We give a possible choice of density functions candidate for the miscellaneous parameters and recall also their prior distribution which were given in the beginning of this section.

1. We have $b_i \sim \mathcal{N}(\mu_i, \Sigma_i)$, $(\sigma_i \sigma_i^*)^{-1} \sim W_p(\alpha_i)$. We define the transition Markov chain as follow.

$$\begin{aligned}\big((b_i)^{m+1}, (\sigma_i)^{m+1}\big) =& \big((b_i^j)^{m+1}, (\sigma_i^{kj})^{m+1}\big)_{k,j\in\{1,\cdot,d\}} \\ =& \big((b_i^j)^m + r_{b_i}^j(\xi_{b_i}^j)^m, (\sigma_i^{kj})^{m+1}\big)\end{aligned}$$

with

$$\begin{aligned}(\sigma_i^{kj})^{m+1} &= (\sigma_i^{kj})^m + r_{\sigma_i}^{kj}(\xi_{\sigma_i}^{kj})^m \\ (\sigma_i^{kk})^{m+1} &= \big|(\sigma_i^{kk})^m + r_{\sigma_i}^{kk}(\xi_{\sigma_i}^{kk})^m\big|\end{aligned}$$

with the $N + N_0$ independent random variables $(\xi_{b_i}^j, \xi_{\sigma_i}^{kj})^m \sim \mathcal{N}(0, I_2)$. Then

$$f_{b_i^j}(x, y) = \mathbb{P}(r_{b_i}^j(\xi_{b^i}^j)_m = y - x) = f_{b_i^j}(y, x),$$

is symmetric, and $f_{\sigma_i \sigma_{i*}}$ also as a combination of Normal and Wishart distributions.

2. $(\mathcal{P})_i \sim Dir(\zeta_{i,1}, \zeta_{i,2}, \zeta_{i,3})$, where $\zeta_{i,j}$ can depend on $(Y_t)_{t\in\{t_0,\ldots,t_n\}}$; $Y_0 \sim \mathcal{U}\{1, 2, 3\}$; $Y_{t_{i+1}} \in \{1, 2, 3\}, \mathbb{P}(Y_{t_{i+1}} = j | Y_{t_i}, \mathcal{P}) = (Y_{t_i}\mathcal{P})_j$. We simulate the new $(\mathcal{P})_i, Y_0$ and $Y_{t_{i+1}}$ from their prior density distributions; again, the transition density is symmetric.

4 Application

4.1 Weekly stock index data

The data in this study consist of weekly returns [1], collected between January 1991 (first week) and December 2011 (third week) for seven stock indexes from developed economies: Germany (DAX), France (CAC40), the United Kingdom (FTSE100), Japan (Nikkei 225), and the United States (S&P500, Nasdaq, and Dow Jones). Denote as the price index at time t (t = 1 . . . 1095). For the sake of simplicity and to obtain percentage values, we define :

$$r_t = 100 \ln(\frac{S_t}{S_{t-1}}) \tag{24}$$

as the compounded return for different stock indexes at time t.
Figures 1 to 7 display time-series plots for the levels and returns of markets indices. Except for the Nikkei and Nasdaq, we observe similar trends. A first rise phase of indices can be seen from early 1990s to the dot-com bubble burst in 2000. Thereafter, we have a decrease phase followed by a turn upward in 2003, which persists until the subprime crisis. This crisis reached its nadir in October 2008 with Lehman Brothers' bankruptcy. The behavior of the Nikkei index is relatively different though, in that during this period, we observe a downward trend in the stock market index. During

[1] E xtracted from http://finance.yahoo.com

1990s, or lost decade, Japanese growth rates were very weak, with an average annual growth of per capita gross domestic product (GDP) of 0.5% [16]. This contraction followed the bursting of the stock market and real estate bubbles at the end of the 1980s.
In addition, the stock returns reveal higher volatility during the financial crisis, between 2000 and 2003 and mainly in 2007/2008. Finally, the Jarque-Bera statistic (Table 1) calculated for the stock return confirms the rejection of the null hypothesis of normality.

(INSERT fig. 1-7) (INSERT table 1)

4.2 Estimating the states

In Figure 8, we present the states for the stock market returns. Because we denote three possible states for the returns, we denote values of 1, 2 and 3 as low, intermediate, and high states. To derive the normalized variables, we subtract the mean and divide by the standard deviation. Several observations may be done:

1. The abnormal returns for US and European markets that is in low and high states do not appear throughout the whole period. For example, for US markets, we obtain abnormal returns during two periods: the dot-com bubble and the subprime crisis. There are few abnormal returns during the 1990s or between 2002 and 2007. It is during the subprime and euro crisis that we observe many abnormal returns for the French index; for the Dax and the CAC40, the abnormal states are fewer during the dot-com crisis.

2. The behavior of the Japanese market is very different. There are many abnormal returns during the 1990s, unlike in the other countries, and fewer abnormal returns between 2000 and 2007. We find more abnormal returns during the subprime crisis, whereas the Japanese stock market does not appear disturbed by the euro crisis.

3. In Table 2, we complete these estimations with the transition matrices for three states. In all cases, the probability of remaining in the intermediate state is very high, around 0.90 for the state and 0.97 for the states. The transition probabilities of abnormal states are lower for the Japanese market (0.22-0.23) than for other stock markets (from 0.30 to 0.45).

Finally, these observations can be completed by calculating of correlations across states.

(INSERT fig. 8) (INSERT table 2)

4.3 Measure of association between states

An important issue for financial markets is the correlation between returns, particularly abnormal returns. Generally, cross-asset correlations may be analyzed using the covariance of returns. Covariance between two assets/market indices is the product of three terms: the volatilities of the two market indices and their correlation. Insofar as the "covariance may vary, not because the correlation between the two assets change, but simply because their individual change" [7], economists and statisticians tend to choose to estimate correlation matrices. But here, we are interested in different states of markets, represented by qualitative variables that can take the three values we have noted. Therefore, a correlation coefficient is not relevant, and we instead retain a contingency table analysis.

First, we characterize the significance of the association between states of returns using a chi-square statistic. However, a test of the null hypothesis of no association between two variables is not sufficient to quantify the strength of association. Several measures of association, analogous to correlation coefficients, appear in prior literature. We adopt the more general measure that is the Cramer's V. This coefficient is bounded between 0 and 1, such as it equals 0 when there is no association and 1 if the association is perfect. We use I and J to denote the number of rows and columns, and N is the total number of events, such as the formula for Cramer's V is: $V = \sqrt{\chi^2/Nmin(I-1, J-1)}$.In Table 3, we present the association measures for states and for three periods: the global period, the dot-com bubble period (May 1994-September 2002) and the subprime and euro crisis period (July 2007-December 2011). For the whole period, we find higher coefficients, around 0.40, among European markets, i.e. FTSE, DAX and CAC indices, and lower coefficients, around 0.10-0.20, between the Nasdaq and these three European markets. When we consider the Nikkei, the null hypothesis of no association cannot be rejected in three cases, that is, in connection with the Nasdaq, FTSE, and S&P500. But this global period covers both a period of strong market increases, that is, the dot-com bubble, and a crisis period as sparked by the subprime episode. Therefore, we calculate statistics for the two sub periods, 1994-2002 and 2007-2011.

During the former dot-com bubble period, we find higher coefficients/associations between European markets, with coefficients between 0.36 and 0.46, whereas the coefficients between US markets and European markets are lower, between 0.14 for the DJ/DAX and 0.37 for the CAC/S&P500. In addition, we note that the association is not different from 0 between the Nikkei and all other markets except the Dow Jones (0.14).

In contrast, the subprime crisis period presents several different features. First, the tests suggest rejecting the null hypothesis of no association. Second, the "correlations" among European markets are very high, including 0.56 between DAX and CAC, and 0.50 among theses two markets indices and FTSE. Third, the association coefficients between the Nikkei and other indices are no longer null with values between 0.20 and 0.32 with the DAX, CAC, FTSE, and DJ, and around 0.14 with the Nasdaq and S&P500.

(INSERT table 3)

5 Conclusion

We propose a method to detect the abnormal states in stock market returns. We study seven indices, over the period January 1991 (first week) – December 2011 (third week). We estimate three possible states: a high rate of return and a low return, both to which constitute abnormal returns with high volatility, and an intermediate state. To determine the state of the markets at each date, we have estimated the parameters of the return process using MCMC method (Metropolis-Hastings algorithm). With Cramer's V coefficient, we deduce the association coefficients or "correlations" among the different states of the major stock exchange markets around the world. First, we show that the associations are globally stronger during the subprime crisis than during the dot-com bubble period. Second, among European markets, we obtain higher Cramer's V, regardless of the period. Third, the association between the Nikkei and the other markets indices is systematically lower, confirming a relative disconnection of the Japanese market from other developed economy markets.

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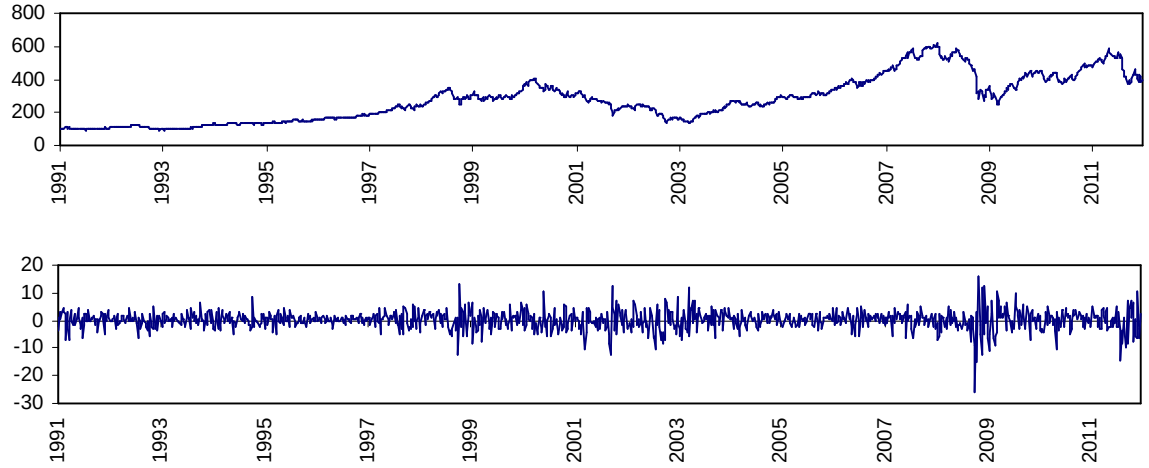

Figure 1: DAX quotes (top) and weekly returns (bottom) 12/31/1991-12/19/2011

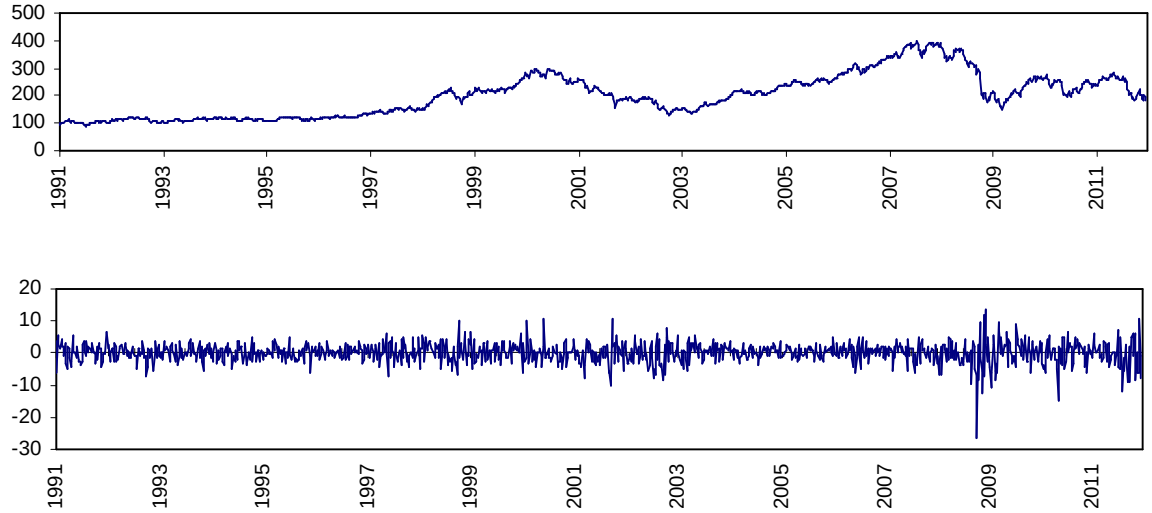

Figure 2: CAC40 quotes (top) and weekly returns (bottom) 12/31/1991-12/19/2011

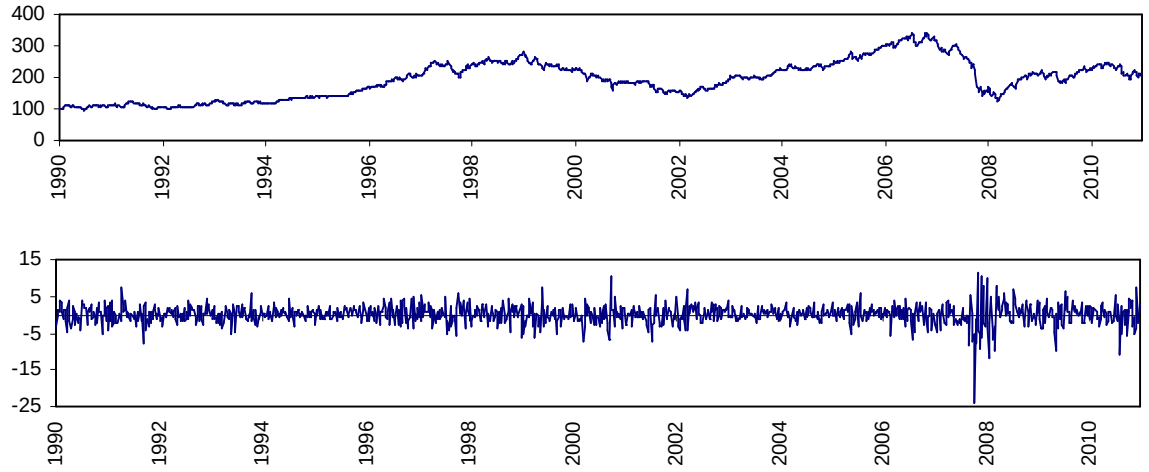

Figure 3: FTSE quotes (top) and weekly returns (bottom) 12/31/1990-12/19/2011

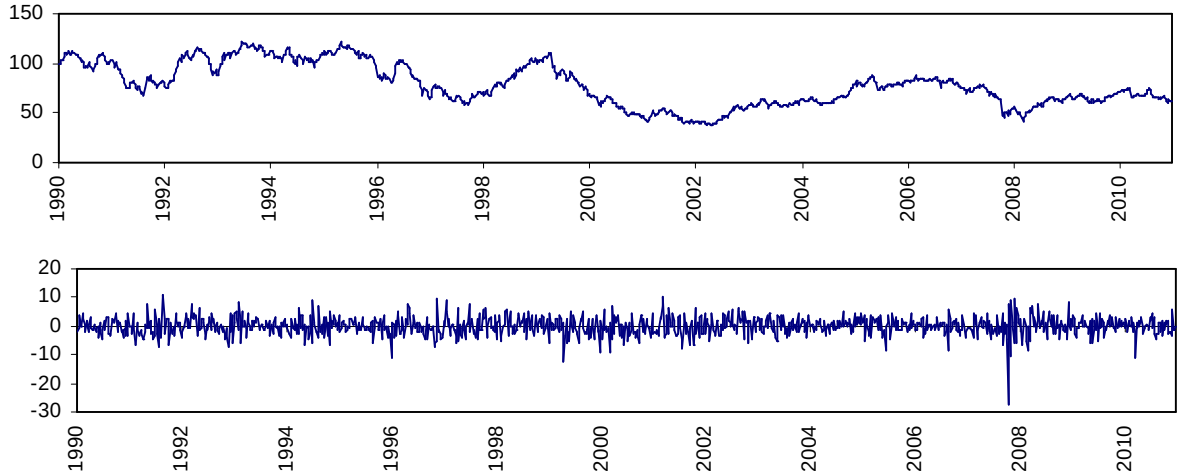

Figure 4: NIKKEI quotes (top) and weekly returns (bottom) 12/31/1990-12/19/2011

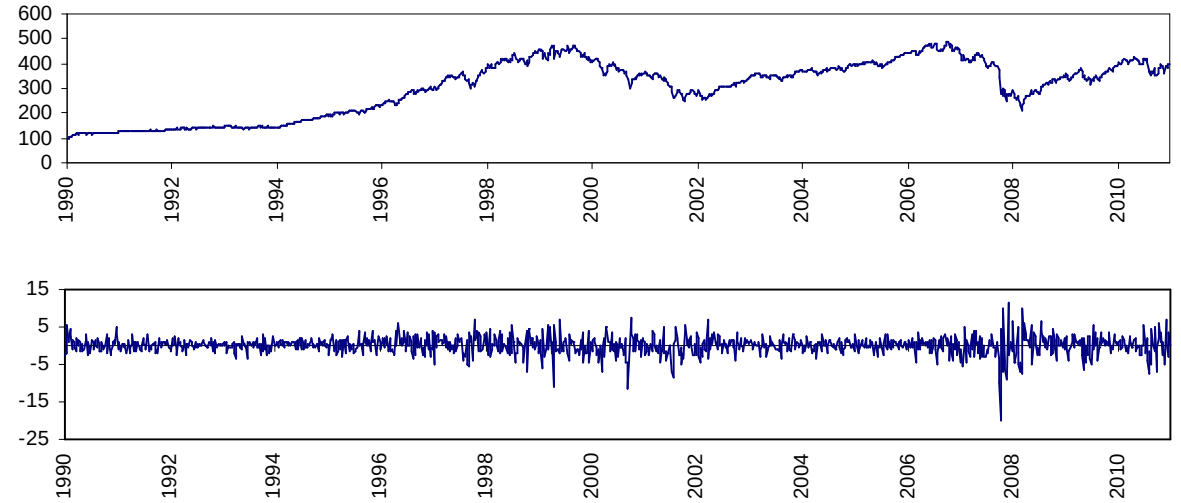

Figure 5: S&P500 quotes (top) and weekly returns (bottom) 12/31/1990-12/19/2011

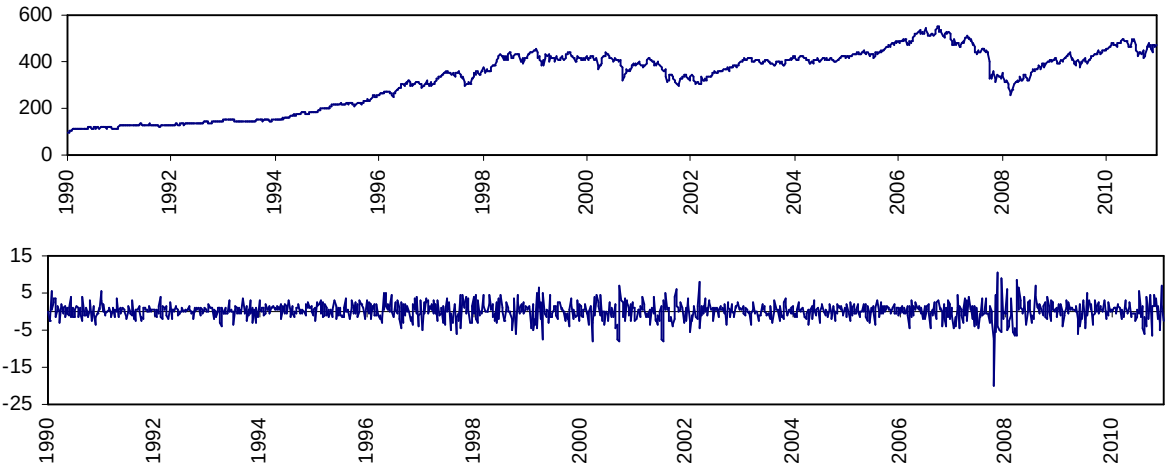

Figure 6: Dow Jones quotes (top) and weekly returns (bottom) 12/31/1990-12/19/2011

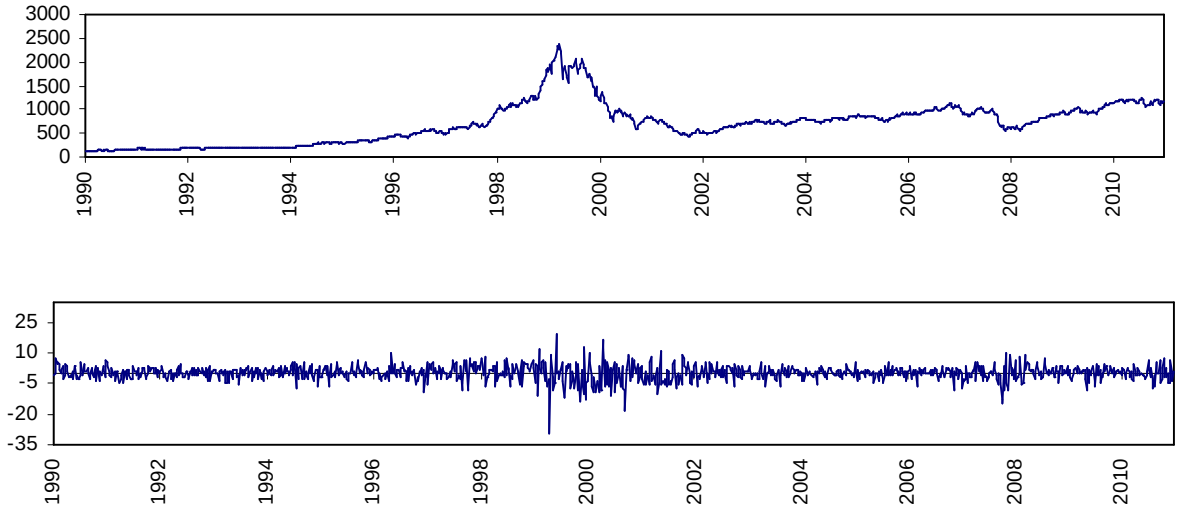

Figure 7: Nasdaq quotes (top) and weekly returns (bottom) 12/31/1990-12/19/2011

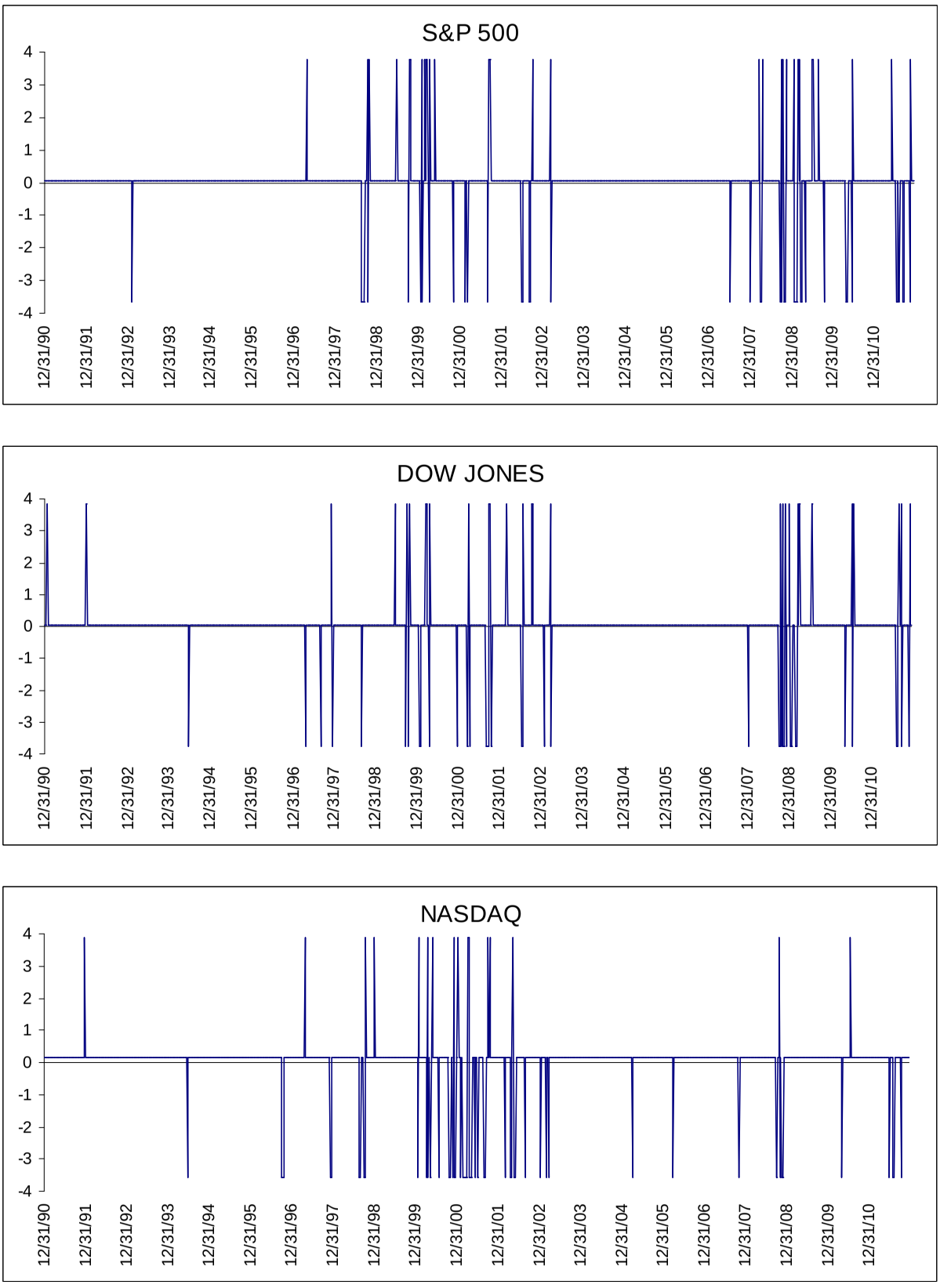

Figure 8 : Estimation of abnormal states

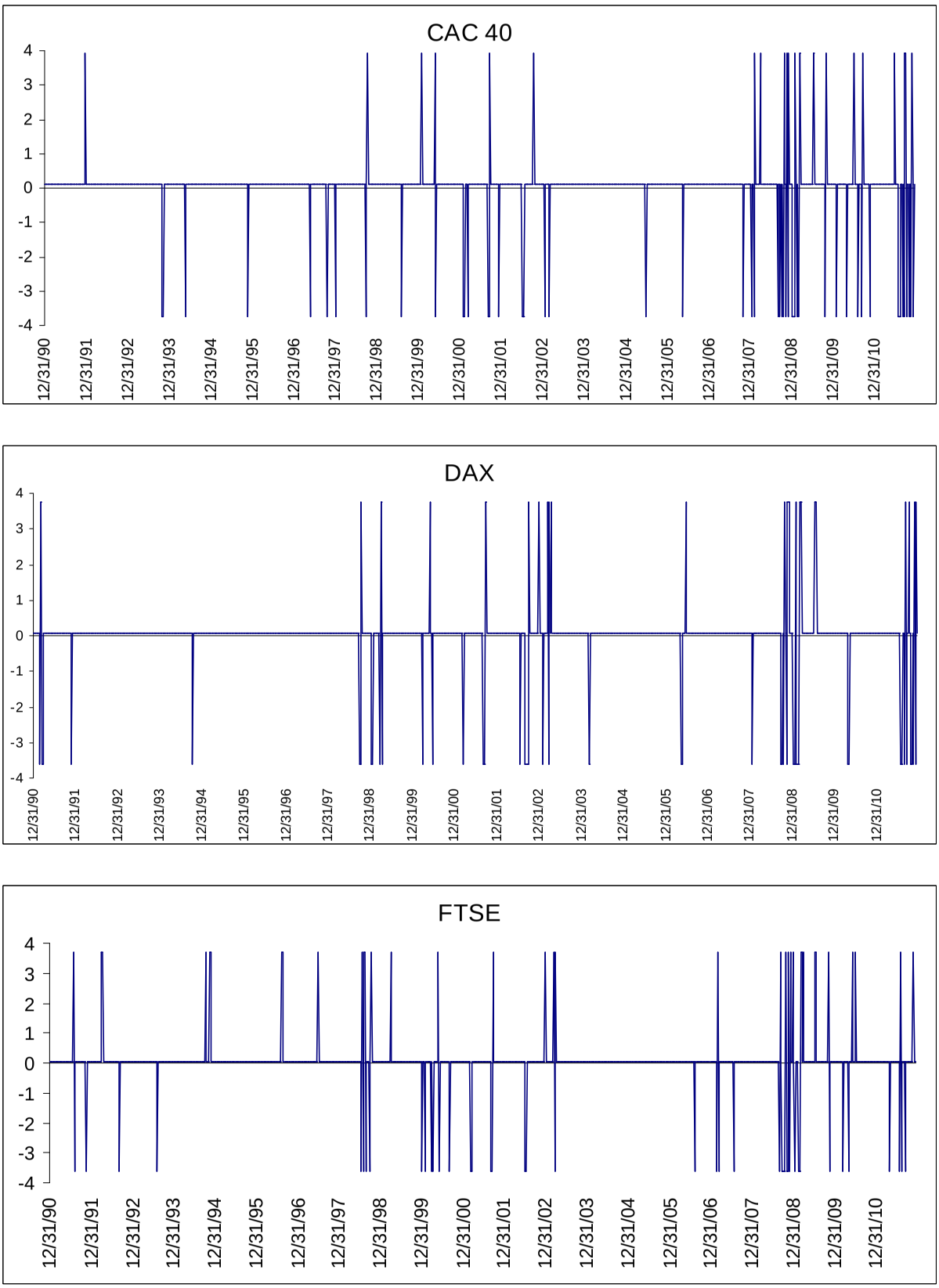

Figure 8: Estimation of abnormal states (continued)

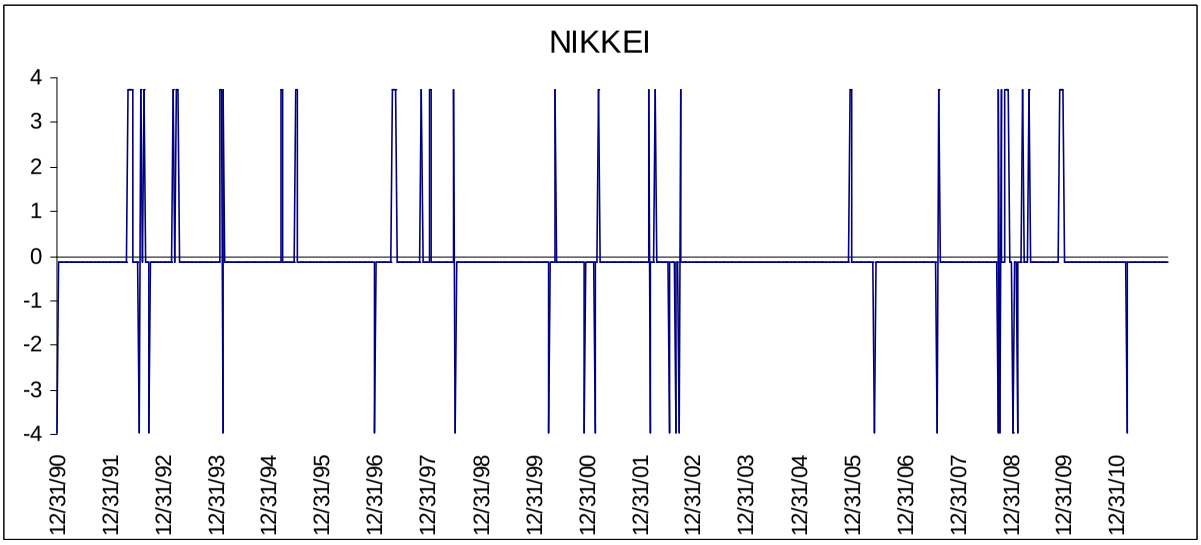

Figure 8: Estimation of abnormal states (continued)

Table1 : Summary statistics of stock returns (x100)
Period 12/31/1990-12/25/2011

Statistic	DAX	CAC40	FTSE	NIKKEI	S&P500	Dow Jones	Nasdaq
Mean	0.1275	0.0617	0.0662	-0.0464	0.1231	0.1374	0.2219
Min	0.4510	0.2917	0.2075	-0.0382	0.2308	0.2717	0.3813
Max	15.7056	13.6505	11.2940	11.1067	11.3559	10.7449	19.1400
Median	-25.6135	-26.3169	-24.0127	-27.4312	-20.0837	-19.8692	-29.1008
Std. Dev.	3.3519	3.1616	2.5966	3.1897	2.4125	2.3299	3.7327
Skewness	-0.6078	-0.7030	-0.8640	-0.5456	-0.7305	-0.6648	-0.6193
Kurtosis	8.1653	8.7015	11.35396	8.2784	9.6639	9.0267	8.3408
Jarque-Bera Test (a)	1284.73* (0.000)	1573.35* (0.000)	3320.35* (0.000)	1325.54* (0.000)	2123.55* (0.000)	1737.85* (0.000)	1371.41* (0.000)
Obs.	1095	1095	1095	1095	1095	1095	1095

(a) p-values are in parentheses. * indicates the rejection of the null hypothesis of a normal distribution

Table 2: Transition Matrices of States

		Low State 1	Int. State 2	High State 3
Dow Jones	S1	0.3600328	0.3922399	0.2477273
	S2	0.0543472	0.9078343	0.0378185
	S3	0.3500569	0.3266098	0.3233333
SP 500	S1	0.4117325	0.3760589	0.2122086
	S2	0.0501945	0.9069498	0.0428557
	S3	0.2844652	0.3443377	0.3711971
Nasdaq	S1	0.4286232	0.3271078	0.2442690
	S2	0.0723289	0.9067297	0.0209414
	S3	0.3091477	0.4306618	0.2601905
FTSE	S1	0.3097893	0.3901177	0.3000930
	S2	0.0529152	0.9053966	0.0416882
	S3	0.2548288	0.3616358	0.3835354
NIKKEI	S1	0.2244978	0.5685314	0.2069708
	S2	0.0269065	0.9149763	0.0581172
	S3	0.2109083	0.2309196	0.5581721
CAC	S1	0.3184206	0.4178978	0.2636816
	S2	0.0780773	0.8912417	0.0306810
	S3	0.3374663	0.4571611	0.2053726
DAX	S1	0.4191528	0.3240655	0.2567818
	S2	0.0564527	0.9119077	0.0316397
	S3	0.2829299	0.3923040	0.3247660

Note: S1, Low State; S2, Intermediate State; S3, High State.

Table 3: Measure of association between states of markets: Cramer's V

Global period: 12/31/1990-12/25/2011							
Index	DJ	S&P500	NASDAQ	FTSE	NIKKEI	DAX	CAC
DJ							
S&P500	0.2311						
NASDAQ	0.2193	0.1750					
FTSE	0.2492	0.2380	0.1622				
NIKKEI	0.1395	0.0782*	0.0760*	0.0851*			
DAX	0.2567	0.2506	0.1344	0.4020	0.1884		
CAC	0.2393	0.2070	0.1917	0.3951	0.1260	0.4269	
Dot-com bubble period: 05/12/1994 – 09/30/2002							
Index	DJ	S&P500	NASDAQ	FTSE	NIKKEI	DAX	CAC
DJ							
S&P500	0.2442						
NASDAQ	0.2275	0.2515					
FTSE	0.1715	0.2500	0.1269				
NIKKEI	0.1431	0.0836*	0.1010*	0.0379*			
DAX	0.1435	0.3202	0.1041*	0.3633	0.1572		
CAC	0.1873	0.3755	0.2020	0.4151	0.0827*	0.4641	
Subprime and euro crisis period: 07/09/2007 – 12/25/2011							
Index	DJ	S&P500	NASDAQ	FTSE	NIKKEI	DAX	CAC
DJ							
S&P500	0.2279						
NASDAQ	0.2756	0.1548					
FTSE	0.3747	0.2573	0.3137				
NIKKEI	0.2020	0.1448	0.1413	0.2278			
DAX	0.4038	0.2943	0.2903	0.4941	0.3231		
CAC	0.2922	0.1903	0.3050	0.4943	0.2382	0.5625	

Note: * indicates that the null hypothesis, i.e. independence between the variables/states, is not rejected (χ^2 statistic) at the 5% level.