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Insurance Pools for New Undiversifiable Risk*

David Alary[†] Catherine Bobtcheff[‡] Carole Haritchabalet[§]

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Abstract

The European insurance industry benefits from some special antitrust exemptions. Indeed, insurers can syndicate, via a "pool", for the coverage of undiversifiable risks. We show that the pool issue amounts to share a common value divisible good between capacity constrained agents with a reserve price and private information. We characterize the equilibrium risk premium of this game and the resulting insurance capacity offered. We then compare the pool to a discriminatory auction upon two dimensions, the total capacity insured and the premiums. There is no clear domination of one auction format. Strength of affiliation and competition are key variables.

Keywords: insurance, competition, undiversifiable risk, common value auction

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1 Introduction

The European insurance industry benefits from some special antitrust exemptions.¹ The European Commission adopted a Block Exemption Regulation in 1992 and when this Regulation expired at the end of March 2010, the Commission, after in-depth consultation with the insurance sector, consumers organizations and public sector bodies adopted a new Insurance Block Exemption Regulation which is valid for seven years. The new insurance Block Exemption Regulation applies Article 101(3) of the European Commission Treaty to grant an exemption to the application of competition rules to certain types of agreements in the insurance sector. One of these agreements concerns the opportunity of insurers to syndicate, via a pool, for the coverage of undiversifiable risks. On 13 December 2016 the Commission took note of the expiry of the IBER on 31 March 2017.

During the consultation, the adequacy of pools has been discussed by the European Commission. What is the impact of this form of cooperation on the demand and supply of insurance and on the pricing of insurance policies? To what extent is the performance (profitability, solvency) of the insurance industry affected? The EC Commission was arguing that such syndicates cannot allow for competitive offers because it favors collusive practices. It also claimed that pools may constitute barriers to entry for new insurers. Insurance companies put forward these arguments. They claim that without pools, there would be a decrease in insurance capacity and thus less protection for new risks.² In particular, they recall that pools for nuclear risks (Assuratome) or medical liability risks (GTAM) were the only solution to provide insurance for those risks. Also, they argue that pools enable insurance companies to share knowledge and experience about certain less frequently occurring risks, which should, according to them, benefit to both sides of the market. The objective of the paper is to analyze the efficiency of such pools. We want to study whereas the exemption for the setting up and operation of insurance pools or the coverage of new risks is justified.

The setting up of a pool can be described as a two-stage game. The first stage is an auction designed for all insurers. An insurer that participate to the auction announces a risk premium and a capacity. Insurers are capacity constraints so that a single insurer

¹See Faure and Van den Bergh (1995) for a description of these exemption clauses.

²Defined in the Regulation as "risks which did not exists before, and for which insurance cover requires the development of an entirely new insurance product not involving an extension, improvement or replacement of an existing insurance product.

cannot take the whole risk. The pool risk premium is the lowest risk premium announced by the insurers. Also, this offer defines a pool leader as the insurer having announced the lowest risk premium. In a second stage, the pool leader is in charge to collect all the capacities up to the policyholders demand at the pool risk premium. Insurers are assumed to be expert in the evaluation of such new risks. They then participate to the auction with a private evaluation of the actuarial premium. This amounts to study the problem of sharing a common value divisible good between capacity constrained agents with a reserve price and private information. The game we consider is then a particular auction of a divisible good with common value and possibly affiliated signals. The first stage is a uniform price auction with an exit option for the followers, and the second stage offers a re-entry option for followers. We characterize the equilibrium risk premium of this game and the resulting insurance capacity offer. We then compare the outcome of the pool to the outcome of discriminatory auctions that are traditionally used to allocate such divisible goods. We next examine how a re-entry option impacts the discriminatory auction.

From now, the literature on undiversifiable risks has focused on the risk sharing problem between insurers and policyholders. This risk sharing problem is analyzed for instance in Doherty and Dionne (1993) or Mahul and Wright (2003). Doherty and Dionne (1993) introduce a new form of insurance contract called Decomposed Risk Transfer contract (DRT contract) defined by an insurance policy packaged with a residual claim on the insurance pool. They show that this contract increases policyholders welfare. They characterize the optimal coverage and the risk premium as a function of the cost of risk bearing derived from asset pricing models. Our setting builds on this contract in the sense that a pool offers a two dimensional contract (a risk premium and a coverage) for which they pay the risk bearing cost. In our model, the pool risk premium (paid by the policyholders) may differ from the actuarial premium (paid by the insurer) because of the particular competition emerging from the pool.

The literature on undiversifiable risks has mainly focused on the risk sharing problem between insurers and policyholders leaving aside the problem of risk sharing between insurers. We think that it is necessary to open the door to other fields of economics to answer this question. Pool agreements are a particular strategic interaction between insurer. Auction theory is then a powerful tool to understand insurance cooperation. This

theory has been widely used since its development by Milgrom (1981).³ We establish a relation between agreeing on a common coverage of a risk and exchanging Treasury debt and other divisible securities (Back and Zender (1993)). Our objective is then to adapt this literature to discuss the costs and benefits of insurance cooperation schemes.

We compare the different auctions format upon two dimensions, the total capacity insured and the premiums. There is no clear domination of one auction format. We show that the strength of affiliation between insurers private information and strength of competition (adjusting capacity constraints) are key variables. Premiums are lower in the uniform auction compared to the discriminatory auction. Allowing for reentry in a second round offers a better insurance coverage. We also show that increasing competition between insurers has two opposite effects on insurance coverage: full coverage of the risk is less likely but the proportion of uninsured risk decreases. Finally, we show that a limited knowledge of risks (a low affiliation between insurers signals) makes the pool more efficient.

The paper is organized as follows. We present the model in section 2. We then solve the equilibrium of the pool in section 3. In section 4, we introduce discriminatory auction and allow followers to re-enter in a second round. Section 5 compares the different outcomes. All proofs are relegated to the appendix.

2 The model

Two identical risk neutral insurers, a and b , are asked for the coverage of an undiversifiable risk. In what follows, i refers to an arbitrary insurer and j to its opponent.

2.1 Risk and insurance contract

In this section, we set up a simple characterization of risk. Consider $n \geq 1$ identical risk averse agents that are exposed to a non diversifiable risk: the aggregate loss nL occurs with probability p .

In the absence of insurance, the expected utility of each agent writes

$$U(p) = pu(w - L) + (1 - p)u(w)$$

where u denotes the increasing and concave utility function and w each agent's initial

³See also Hendricks, Porter and Tan(2008)or Haile (2003).

wealth.

Assume that agents can insure this risk. We define β as the proportion of the risk demanded by agents and P the unit risk premium asked by the insurers to bear this undiversifiable risk. The expected utility of each agent with such a contract writes

$$V(\beta, P, p) = pu(w - L + \beta L - \beta LP) + (1 - p)u(w - \beta LP)$$

where βL is the indemnity paid by insurers in case of loss, and βLP is the insurance premium paid for this coverage. The contract can be then completely defined by (β, P) , the proportion of the loss that is insured and the unit risk premium.

We define $\bar{P}(\beta, p) \equiv \bar{P}$ as the maximum premium that agents are willing to pay for this coverage. At \bar{P} , they are indifferent between insuring or not:

$$V(\beta, \bar{P}, p) = U(p).$$

This minimum risk premium that insurers are willing to accept for this coverage is the actuarial premium rate p . The insurer net expected benefit of such a contract writes

$$\beta L(P - p).$$

The term $L(P - p)$ is the net unit premium received by the insurer once the risk has been transferred to the reinsurance market which in particular means that no indemnity is paid by the insurer to the insuree. We must have $p \leq P \leq \bar{P}$ for the insurance contract to exist.⁴

2.2 Insurers' expertise for new risks

We assume that the probability of the undiversifiable risk p is not perfectly known by the agents. All agents have the same prior on this probability, denoted p_0 (the a priori actuarial premium). This belief defines the *a priori* maximum premium \bar{P}_0 implicitly defined by $V(\beta, \bar{P}_0, p_0) = U(p_0)$.

Insurers are assumed to be expert in the evaluation of such new risks. They have the ability to better identify the true risk. This assumption reflects the fact that insurers often concentrate their activities in specific lines of business and can use their expertise

⁴We have that $\beta = 1$ if and only if $P = p$.

to infer the probability of new risks. Consequently, we assume that insurers can obtain a costless signal related to the true probability. S_i (resp. S_j) is the signal privately observed by insurer i (resp. j). The two signals S_i and S_j whose realizations are denoted s_i and s_j are assumed to be affiliated. They are distributed according to the same continuous distribution on the interval $[0, 1]$. Let $g(\cdot|s)$ denote the (symmetric) probability distribution function of an insurer's signal conditional on the other insurer having observed signal s . This translates into the following assumption on the family of densities $g(\cdot|s)$.

Assumption 1

$$\forall s'_i > s_i \text{ and } s'_j > s_j, \frac{g(s'_i|s'_j)}{g(s'_i|s_j)} \geq \frac{g(s_i|s'_j)}{g(s_i|s_j)}. \quad (1)$$

This implies that the actuarial premium rate is a function of insurers' private information. While it is *ex ante* unknown to any particular insurer, it is *ex post* common to all insurers. We consider that this function is the same for the two insurers and that it can be expressed as a symmetric function of all insurers' signals.

$$p(s_i, s_j) = p(s_j, s_i). \quad (2)$$

We impose the following regularity assumptions on the actuarial premium rate.

Assumption 2 *The actuarial premium rate p satisfies the following properties.*

- (i) *Function p is twice continuously differentiable and strictly increasing in the two variables;*
- (ii) $\mathbb{E}[p(S_i, S_j)|S_j = 0] < \bar{P}_0 < \mathbb{E}[p(S_i, S_j)|S_j = 1]$;
- (iii) $s \mapsto p(s, s)$ *is convex;*
- (iv) *Function p is supermodular,*

$$\frac{\partial^2 p(s_i, s_j)}{\partial s_i \partial s_j} \geq 0, \forall (s_i, s_j) \in [0, 1]^2. \quad (3)$$

Observe that a high value of s signals a risk that is assumed to be more costly to insure and that some risks cannot be insured. Assumption 2(ii) means that if insurer j observes the best (resp. worst) possible signal, covering the risk is always (resp. never) profitable for insurer i .

Definition 1

(i) $\tilde{\sigma}$ is implicitly defined by

$$p(\tilde{\sigma}, \tilde{\sigma}) = \bar{P}_0. \quad (4)$$

(ii) α is implicitly defined by

$$p(\alpha(x), x) = \bar{P}_0 \quad \forall x \in [0, 1]. \quad (5)$$

Signal $\tilde{\sigma}$ is the maximal signal for which the two insurance companies accept to cover the risk in case they observe the same signal. Function α can be interpreted as an isocost curve evaluated at the maximal premium \bar{P}_0 .⁵

Given our assumptions, if P is paid by insurees, insurer i 's net expected benefit of providing a unit coverage writes⁶

$$(P - \mathbb{E}[p(S_i, S_j) | S_i = s_i]). \quad (6)$$

Insurers compete for the coverage of this risk by choosing the quantity they insure and the price at which they provide insurance. The existence of solvency regulation and capital requisites for the coverage of such new and undiversifiable risk implies that insurers are capacity constrained. Therefore, a single insurer can not offer more than a proportion $\bar{\beta}_i \leq \beta$, with $\bar{\beta}_i = \bar{\beta}_j = \bar{\beta}$. We assume that the market is too small to absorb the full capacity of the two insurers, i.e. $\beta \leq 2\bar{\beta}$.⁷ To measure the strength of competition on the insurance market, we define

$$\kappa = \frac{2\bar{\beta} - \beta}{\bar{\beta}} \in [0, 1], \quad (7)$$

which can be interpreted as the relative excess supply. When $\kappa = 0$, the two insurance companies may sell their entire capacity so that there is no competition. On the contrary, when $\kappa = 1$, a unique insurer could satisfy the whole demand leading to intense competition. Note that equation (7) implies that the proportion of the total demand an insurer can satisfy by itself, $\bar{\beta}/\beta$, equals $1/(2 - \kappa)$.

These capacity constraints require to organize insurance supply in a syndicated form.

⁵According to Assumption 2(ii), α is a decreasing function. Moreover, the symmetry of α with respect to its arguments implies that $\alpha^{-1} = \alpha$.

⁶In what follows, we normalize L to 1.

⁷Assuming sufficiently large capacities can avoid a monopoly outcome and allows to focus on the most interesting cases.

2.3 Insurers' syndication

The organization of insurance supply amounts to the problem of sharing a common value divisible good between capacity constrained agents with a reserve price. Typically, this issue has been addressed for the particular case of Treasury Bonds. These bonds are usually exchanged through a uniform auction or a discriminatory auction. The insurance industry has its own practice to provide coverage for undiversifiable risks under capacity constraints. Such agreements are named "pools". Even if these different auctions have specific rules, they all share the following timing.

1. Each insurer performs a risk analysis and receives a private signal s_i .
2. If it decides to participate⁸, it announces a risk premium $P_i \leq \bar{P}_0$ based on its own signal.⁹
3. The insurer that announces the smallest risk premium, $P_L = \min(P_a, P_b)$, becomes the leader of the syndicate, the other insurer is the follower.
4. The leader has the priority to choose the capacity it wants. Given the linearity of the insurer's expected net benefit (see Equation (1)), the leader (resp. the follower) sells its full capacity $\bar{\beta}$ (the remaining capacity $\beta - \bar{\beta}$) in case of an exchange.
5. If the two insurers announce the same risk premium, they share the capacity so that they both sell $\beta/2$ at the same price.
6. The terms of exchange depend on the syndicate's format.

The objective of this paper is to analyze different auction rules to constitute the syndicate, namely the pool and the more standard discriminatory auction. Each auction determines a game of incomplete information among the insurers: we look for a symmetric Bayesian Nash equilibrium that is increasing in the bidding strategies of each resulting game.

⁸Insurance is provided only if at least one insurer accepts to participate.

⁹Recall that \bar{P}_0 is the maximum premium that policyholders are willing to pay.

3 Analysis of the pool

3.1 Description of the pool

Ernst and Young (2014) provides a detailed description of the procedures leading to agreements in several European countries. Even if some country-specific differences exist, they share some common features. The pool premium P^P is unique and equals the lowest bid: this is a uniform auction. Ernst and Young (2014) also notes that “the followers are usually invited to either accept or decline or take a share of the risk on the same terms and conditions as the lead insurer”.

We summarize these features with the following rules.

- A first price auction determines the pool risk premium.
- If the two insurers submit a bid $P_i \leq \bar{P}_0$,
 - o The pool risk premium is $P^P = \min(P_a, P_b)$. The pool leader sells $\bar{\beta}$ at price P^P .
 - o The follower observes P^P and decides whether it withdraws from the pool or not. If it does not, it sells $\beta - \bar{\beta}$ at price P^P .
- If only insurer i submits a bid $P_i \leq \bar{P}_0$,
 - o The pool risk premium is $P^P = P_i$. The pool leader sells $\bar{\beta}$ at price P^P .
 - o Insurer j observes P^P and decides whether it enters into the pool or not. If it does, it sells $\beta - \bar{\beta}$ at price P^P .

This particular syndication works as if there exists two rounds. The pool’s rule states that the follower can join or quit the pool whatever its initial choice to submit a bid. A player submits a bid only if it is sufficiently optimistic about the new risk ex ante. If it turns out to be the follower, it has always the possibility to quit the pool after having observed P^P . Also, if a player is too pessimistic to submit a bid ex ante, it may still, in a second round, re-enter and participate if the leader’s bid reveals a good risk.

3.2 Separating equilibrium

We first look for an equilibrium in strictly increasing and symmetric bidding strategies that are characterized by a threshold $\hat{\sigma}^P$ such that

- when $s_i \leq \hat{\sigma}^P$, firm i bids according to a strictly increasing bidding strategy $P^P(s_i)$ with $P^P(\hat{\sigma}^P) = \bar{P}_0$,
- when $s_i > \hat{\sigma}^P$, firm i is willing to participate in the second round only.

In such a separating equilibrium, the bid a insurer submits unambiguously reveals the signal it observes. The profit of firm i that observed a signal s_i and bids a risk premium $P^P(b)$ reads¹⁰

$$\Pi^P(b, s_i) = \begin{cases} \bar{\beta} \int_b^1 (P^P(b) - p(s_i, s_j)) g(s_j|s_i) ds_j \\ \quad + (\beta - \bar{\beta}) \int_0^b (P^P(s_j) - p(s_i, s_j))_+ g(s_j|s_i) ds_j & \text{for } b \leq \hat{\sigma}^P \quad (8a) \\ (\beta - \bar{\beta}) \int_0^{\hat{\sigma}^P} (P^P(s_j) - p(s_i, s_j))_+ g(s_j|s_i) ds_j & \text{for } b > \hat{\sigma}^P \quad (8b) \end{cases}$$

Let us explain the different terms composing the expression of $\Pi^P(b, s_i)$.

- When firm i bids $P^P(b) \leq \bar{P}_0$ (equation (8a)), two situations may arise:
 1. Firm j is only willing to participate in the second round or firm j participates and proposes a risk premium greater than $P^P(b)$. Firm i ' turns out to be the pool leader and serves $\bar{\beta}$ at its proposed price $P^P(b)$;
 2. Firm j participates and proposes a risk premium smaller than $P^P(b)$. Firm i ' turns out to be the pool follower and, in case it is profitable, serves the remaining capacity $\beta - \bar{\beta}$ at firm j proposed price $P^P(s_j)$.
- When firm i observes a signal strictly greater than $\hat{\sigma}^P$ it is only willing to participate in the second round (equation (8b)). Two situations may arise.
 1. Firm j bids a risk premium strictly less than \bar{P}_0 . Firm i , as the pool follower, participates only in case it is profitable and serves the remaining capacity $\beta - \bar{\beta}$ at firm j proposed price $P^P(s_j)$.
 2. Firm j is also only willing to participate in the second round meaning that it observed a signal greater than $\hat{\sigma}^P$, too. No trade will occur for any player.

¹⁰Note that it is independent of i since the strategies are the same for the two insurers.

Incentive compatibility requires that bidders with signals greater than $\hat{\sigma}^P$ prefer not to bid to submitting the bid \bar{P}_0 .

$$\int_{\hat{\sigma}^P}^1 (\bar{P}_0 - p(\hat{\sigma}^P, s_j)) g(s_j | \hat{\sigma}^P) ds_j = 0. \quad (9)$$

Lemma 1 *The threshold $\hat{\sigma}^P$ exists and is unique. Moreover, $\hat{\sigma}^P < \tilde{\sigma}$.*

Given the specific rules of the pool (uniform pricing, options to exit and to re-enter), the follower profit is the same whatever the initial choice to participate to the auction. Therefore, the threshold is only determined by the condition that the leader's expected profit is non negative which refrains from bidding when signals are too high.

At equilibrium, $P^P(b) = P^P(s_i), \forall s_i \leq \hat{\sigma}^P$ so that

$$\frac{\partial \Pi^P(b, s_i)}{\partial b} \Big|_{b=s_i} = 0, \forall s_i \leq \hat{\sigma}^P.$$

This implies that the equilibrium bid $P^P(s_i)$ satisfies the following differential equation

$$P^{P'}(s_i) = \kappa \frac{g(s_i | s_i)}{1 - G(s_i | s_i)} (P^P(s_i) - p(s_i, s_i)). \quad (10)$$

The differential equation (10) is solved with the boundary condition that $P^P(\hat{\sigma}^P) = \bar{P}_0$. In order the bidding strategy to be strictly increasing, a necessary condition is that $\hat{\sigma}^P \leq \tilde{\sigma}$ (Lemma 1). We then obtain the following equilibrium strategy.

Proposition 1 *There exists a unique symmetric Nash equilibrium in strictly increasing bidding equilibrium strategies where*

$$P^P(s) = \bar{P}_0(1 - L(\hat{\sigma}^P | s)) + \int_s^{\hat{\sigma}^P} p(x, x) dL(x | s) \quad \forall s \leq \hat{\sigma}^P \quad (11)$$

with

$$L(x | s) = 1 - \exp \left(-\kappa \int_s^x \frac{g(\tau | \tau)}{1 - G(\tau | \tau)} d\tau \right) \quad (12)$$

and where $L(x | s)$ is an increasing function with $L(s | s) = 0$ and $L(1 | s) = 1$.

Insurer i 's equilibrium expected profit thus writes as

$$\Pi^{P^*}(s_i) = \begin{cases} \int_{s_i}^1 \bar{\beta} (P^P(s_i) - p(s_i, s_j)) g(s_j|s_i) ds_j \\ \quad + \int_0^{s_i} (\beta - \bar{\beta}) (P^P(s_j) - p(s_i, s_j))_+ g(s_j|s_i) ds_j & \text{for } s_i \leq \hat{\sigma}^P \quad (13a) \\ \int_0^{\hat{\sigma}^P} (\beta - \bar{\beta}) (P^P(s_j) - p(s_i, s_j))_+ g(s_j|s_i) ds_j & \text{for } s_i > \hat{\sigma}^P \quad (13b) \end{cases}$$

Figure 1 presents insurer i 's profit as a function of insurers' signals.

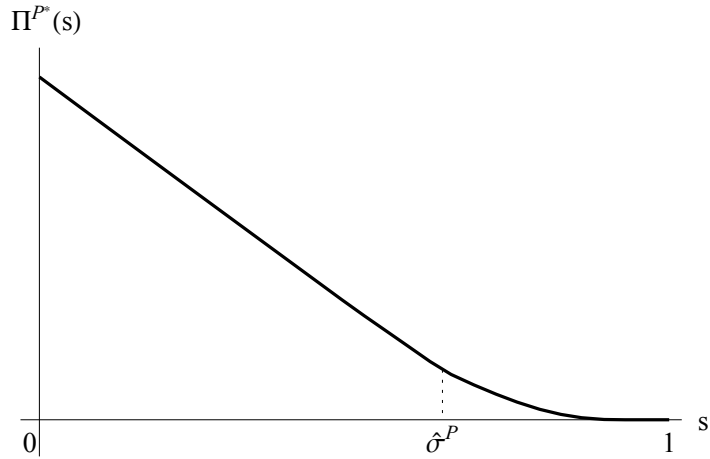


Figure 1: $\Pi_i^{P^*}(s_i)$.

One of the specificities of the pool is not only that the insurer may want to enter in the second round when it observes a signal $s_i > \hat{\sigma}^P$, but also that it may decide not to participate to the auction (ex-post) if its payoff is negative when it receives a signal $s_j \leq s_i \leq \hat{\sigma}^P$ (and turns out to be the follower). This happens when $P^P(s_j) < p(s_i, s_j)$ with $s_j < s_i$. In this case, the capacity is not fully served. The following lemma tells us that this will never be the case: even the most pessimistic insurer never wants to withdraw ex-post.

Lemma 2 *When participating to the pool, the follower never withdraws ex-post:*

$$P^P(s_j) - p(s_i, s_j) > 0 \quad \forall s_j < s_i \leq \hat{\sigma}^P. \quad (14)$$

The specific rules applying to the follower makes the leader position less enviable. The tradeoff is between insuring a large capacity with the risk of ex-post negative profit and insuring a smaller capacity at no risk of loss. This refrains the leader from bidding for high

signal values. This result also holds when s_i is greater than but close to $\hat{\sigma}^P$ (continuity). However, when its signal is too high it may not want to enter as Figure 2 illustrates.

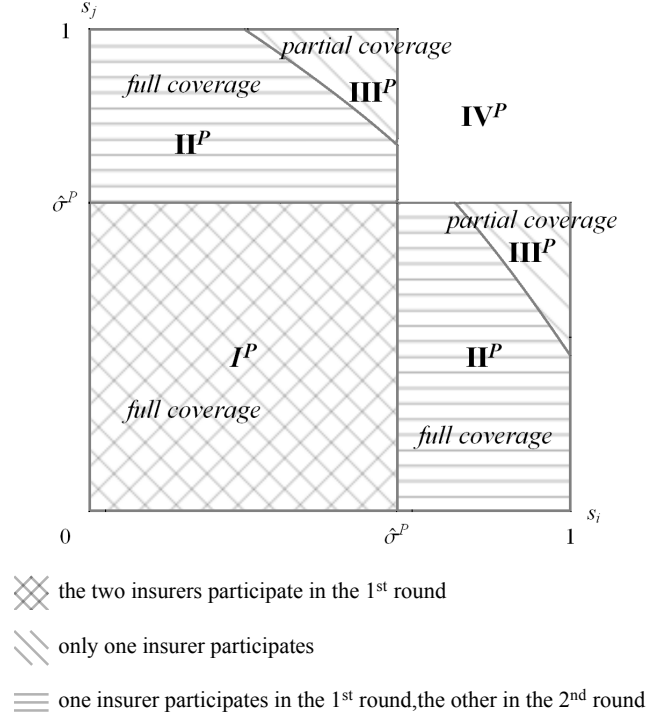


Figure 2: The different regions.

Let us describe Figure 2 assuming insurer i 's signal s_i is the smallest and that it is smaller than $\hat{\sigma}^P$:

- in region I^P , $s_i \leq s_j \leq \hat{\sigma}^P$: both insurance companies bid in the first round and never withdraw, total capacity is insured;
- in region II^P , $s_i \leq \hat{\sigma}^P$, $s_j \geq \hat{\sigma}^P$ and $P^P(s_i) > p(s_i, s_j)$: insurer i bids in the first round and insurer j enters in the second round, total capacity is insured;
- in region III^P , $s_i \leq \hat{\sigma}^P$, $s_j \geq \hat{\sigma}^P$ and $P^P(s_i) < p(s_i, s_j)$: insurer i bids in the first round and insurer j does not participate to the pool, only the leader provides capacity $\bar{\beta}$;
- the boundary between regions II^P and III^P is $\{(s_i, s_j) \in [0, 1]^2 | P^P(s_i) = p(s_i, s_j)\}$: insurer j is indifferent between entering in the second round and never participating to the pool;

- in region IV^P , the two insurers observe a signal greater than $\hat{\sigma}^P$, none of them submits a bid and no trade occurs.

Note that the boundary between regions II^P and III^P might be non-monotonic with respect to s_i . In particular, as $\frac{\partial^2 R^P(s_i)}{\partial s_i \partial \kappa} \geq 0$, the higher κ , the steeper $R^P(s_i)$. Therefore, if $s_i \mapsto R^P(s_i) - r(s_i, s_j)$ is a decreasing function of s_i when $\kappa = 0$; it might be a non monotonic function of s_i when κ is close to 1 as the following graph illustrates.

3.3 Impact of the strength of competition

First, observe that the region of the signal values for which insurers decide to submit a bid in the first round is independent of the strength of competition (see equation (9)). Being a follower in the first round or in the second round yields exactly the same (non-negative) profit. Then, $\hat{\sigma}^P$ only matters for the leader's strategy and is determined to guarantee that the **unit** maximum net expected benefit is non negative. As a consequence, $\hat{\sigma}^P$ does not depend on κ and so region I_i^P , where the total capacity is insured. However, the value of κ modifies the equilibrium bid P^P which in turn affects the follower decision to enter or not in the second round (the boundary between regions II_i^P and III_i^P).

Proposition 2 *When competition increases*

- $P^P(s)$ decreases;
- Region II_i^P (resp. III_i^P) shrinks (resp. expands).

The equilibrium bidding strategy is represented in Figure 3 for two values of κ . Competition unambiguously lowers premiums. When competition increases, the pool more often fails in offering complete coverage. Even if region III_i^P expands with κ , the proportion of the risk insured ($\frac{\bar{\beta}}{\beta} = \frac{1}{2-\kappa}$) increases. Therefore, increasing competition has two opposite effects on the coverage : full coverage is less likely but the proportion of uninsured risk decreases.

4 Discriminatory auction

This part is devoted to the analysis of the discriminatory auction, when reentry is possible or not.

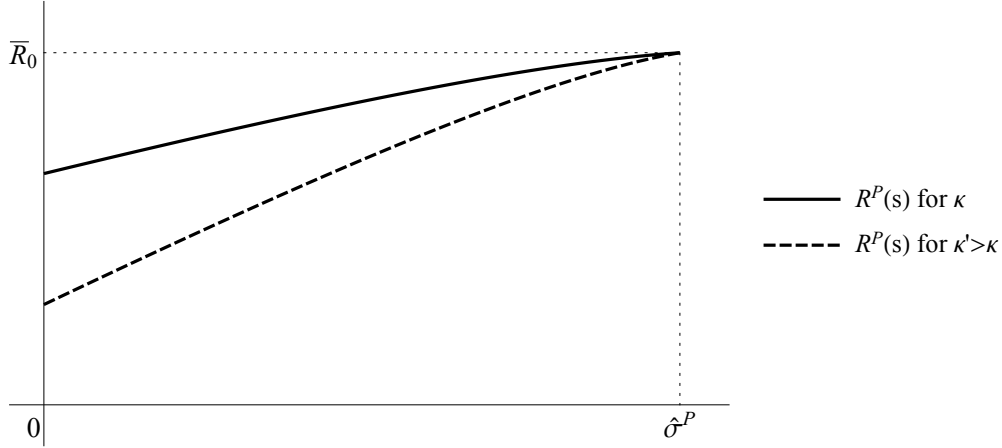


Figure 3: The equilibrium bidding strategy $P^P(s)$ for two values of κ .

4.1 Discriminatory auction without reentry

The bidding process is organized in a single round where each firm proposes a risk premium according to the private signal it received.¹¹ The leader and the follower (if any) sell at their announced risk premium.

4.1.1 Equilibrium analysis

Separating equilibrium. We first look first for an equilibrium in strictly increasing and symmetric bidding strategies. Proceeding as before, assume there exists a threshold $\hat{\sigma}^D$ such that:

- when $s_i \leq \hat{\sigma}^D$, firm i bids according to a strictly increasing bidding strategy $P^D(s_i)$ with $P^D(\hat{\sigma}^D) = \bar{P}_0$,
- when $s_i > \hat{\sigma}^D$, firm i does not participate anymore.

The profit of firm i that observed a signal s_i and bids a risk premium $P^D(b)$ reads

$$\Pi^D(b, s_i) = \begin{cases} \bar{\beta} \int_b^1 (P^D(b) - p(s_i, s_j)) g(s_j | s_i) ds_j \\ \quad + (\beta - \bar{\beta}) \int_0^b (P^D(b) - p(s_i, s_j)) g(s_j | s_i) ds_j & \text{for } b \leq \hat{\sigma}^D \quad (15a) \\ 0 & \text{for } b > \hat{\sigma}^D. \quad (15b) \end{cases}$$

¹¹When the risk is shared with a discriminatory auction without reentry, no specific agreement between the auction participants is needed. In particular, a syndicate is not necessary to organize the trade.

- When firm i bids $P^D(b) \leq \bar{P}_0$ (equation (15a)), two situations may arise.
 1. Firm j does not participate to the auction or participates and proposes a risk premium greater than $P^D(b)$. Firm i turns out to be the syndicate leader and serves $\bar{\beta}$ at its proposed price $P^D(b)$;
 2. Firm j participates and proposes a risk premium smaller than $P^D(b)$. Firm i turns out to be the syndicate follower and serves the remaining capacity $\beta - \bar{\beta}$ at its proposed price $P^D(b)$.
- When firm i observes a signal greater than $\hat{\sigma}^D$, it prefers not to bid (not to participate) to the auction.

Incentive compatibility requires that bidders with signals greater than $\hat{\sigma}^D$ prefer not to bid to submitting the bid \bar{P}_0 .

$$\int_{\hat{\sigma}^D}^1 (\bar{P}_0 - p(\hat{\sigma}^D, s_j)) g(s_j | \hat{\sigma}^D) ds_j + \int_0^{\hat{\sigma}^D} (1 - \kappa) (\bar{P}_0 - p(\hat{\sigma}^D, s_j)) g(s_j | \hat{\sigma}^D) ds_j = 0. \quad (16)$$

Lemma 3 *The threshold $\hat{\sigma}^D$ exists and is unique.*

Contrary to equation (9) that defined the pool threshold, the follower's payoff (the second term of equation (16)) matters. As a consequence, the leader's expected payoff is negative at the threshold making the winners' curse more present. Indeed, an insurer bids until $\hat{\sigma}^D$ in the expectation of being the follower rather than the leader.

As for the pool, the first order conditions imply that the equilibrium bid $P^D(s_i)$ satisfies the following differential equation

$$P^{D'}(s_i) = \frac{\kappa g(s_i | s_i)}{1 - \kappa G(s_i | s_i)} (P^D(s_i) - p(s_i, s_i)), \quad (17)$$

and we must have $\hat{\sigma}^D \leq \tilde{\sigma}$ in order the bidding strategy to be strictly increasing. Whether $\hat{\sigma}^D \leq \tilde{\sigma}$ depends now on the parameters of the model (the shape of both the actuarial premium rate p , the conditional probability distribution characterized by the density probability distribution g and the cumulative probability distribution G and the strength of competition κ).¹²

¹²See Proposition 4 for a comparative static analysis of $\hat{\sigma}^D$ with respect to κ .

Therefore, if $\hat{\sigma}^D \leq \tilde{\sigma}$, the differential equation (17) is solved with the boundary condition $P^D(\hat{\sigma}^D) = \bar{P}_0$.

Semi-pooling equilibrium. If $\hat{\sigma}^D > \tilde{\sigma}$, the equilibrium strategy we just derived is not an equilibrium since it is not strictly increasing (see equation 17). We must look for another equilibrium strategy that involves pooling for some values of the signal. More precisely, we look for an equilibrium in symmetric and increasing bidding strategy that is characterized by two thresholds $\underline{\sigma}^D$ and $\bar{\sigma}^D > \underline{\sigma}^D$ such that

- when $s_i \in [0, \underline{\sigma}^D]$, firm i bids according to a strictly increasing bidding strategy $P^D(s_i)$ with $P^D(\underline{\sigma}^D) = \bar{P}_0$,
- when $s_i \in [\underline{\sigma}^D, \bar{\sigma}^D]$, firm i bids \bar{P}_0 ,
- when $s_i > \bar{\sigma}^D$, firm i does not participate anymore.

The equilibrium is thus separating when $s_i \in [0, \underline{\sigma}^D]$ and it is pooling when $s_i \in [\underline{\sigma}^D, \bar{\sigma}^D]$. The profit of firm i that received a signal s_i and proposes a risk premium $P^D(b)$ reads

$$\Pi^D(b, s_i) = \begin{cases} \bar{\beta} \int_b^1 (P^D(b) - p(s_i, s_j)) g(s_j | s_i) ds_j \\ \quad + (\beta - \bar{\beta}) \int_0^b (P^D(b) - p(s_i, s_j)) g(s_j | s_i) ds_j & \text{for } b \leq \underline{\sigma}^D & (18a) \\ \bar{\beta} \int_{\bar{\sigma}^D}^1 (\bar{P}_0 - p(s_i, s_j)) g(s_j | s_i) ds_j \\ \quad + \frac{\beta}{2} \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(s_i, s_j)) g(s_j | s_i) ds_j \\ \quad + (\beta - \bar{\beta}) \int_0^{\underline{\sigma}^D} (\bar{P}_0 - p(s_i, s_j)) g(s_j | s_i) ds_j & \text{for } \underline{\sigma}^D < b \leq \bar{\sigma}^D & (18b) \\ 0 & \text{for } b > \bar{\sigma}^D & (18c) \end{cases}$$

Contrary to (15), there is a new intermediate case where firm i bids \bar{P}_0 (equation (18b)). Three situations may arise.

1. Firm j does not participate to the auction, firm i is the only participant and serves its capacity $\bar{\beta}$ at price \bar{P}_0 ;
2. Firm j also bids \bar{P}_0 . Firms i and j therefore share the market and each serves a capacity $\beta/2$ at price \bar{P}_0 ;

3. Firm j bids a risk premium strictly less than \bar{P}_0 . Firm i ' turns out to be the syndicate follower and serves the remaining capacity $\beta - \bar{\beta}$ at \bar{P}_0 .

Incentive compatibility requires that insurers with signal in $[\underline{\sigma}^D, \bar{\sigma}^D]$ prefer submitting \bar{P}_0 to not participating and to submitting any lower bid. Moreover, insurers with signals greater than $\bar{\sigma}^D$ prefer not to bid to submitting the bid \bar{P}_0 . The two thresholds are thus defined by the following system.

$$\left\{ \begin{array}{l} \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(\underline{\sigma}^D, s_j)) g(s_j | \underline{\sigma}^D) ds_j = 0 \\ \int_{\bar{\sigma}^D}^1 (\bar{P}_0 - p(\bar{\sigma}^D, s_j)) g(s_j | \bar{\sigma}^D) ds_j + \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} \left(1 - \frac{\kappa}{2}\right) (\bar{P}_0 - p(\bar{\sigma}^D, s_j)) g(s_j | \bar{\sigma}^D) ds_j \\ + \int_0^{\underline{\sigma}^D} (1 - \kappa) (\bar{P}_0 - p(\bar{\sigma}^D, s_j)) g(s_j | \bar{\sigma}^D) ds_j = 0. \end{array} \right. \quad (19a)$$

It must also be checked that an insurer that bids \bar{P}_0 when it observes a signal comprised between $\underline{\sigma}^D$ and $\bar{\sigma}^D$ does not have an incentive to underprice. This comes down to checking that¹³

$$\int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(s_i, s_j)) g(s_j | s_i) ds_j \leq 0 \quad \forall s_i \in [\underline{\sigma}^D, \bar{\sigma}^D].$$

Lemma 4 *The semi-pooling equilibrium exists and is unique if and only if the separating equilibrium does not exist ($\hat{\sigma}^D > \tilde{\sigma}$). Moreover, if the semi-pooling equilibrium exists, the following ranking holds*

$$\alpha(\bar{\sigma}^D) \leq \underline{\sigma}^D < \tilde{\sigma} < \alpha(\underline{\sigma}^D) \leq \bar{\sigma}^D \leq \hat{\sigma}^D.$$

When the separating equilibrium exists ($\hat{\sigma}^D < \tilde{\sigma}$), the system ((19a)-(19b)) has a unique solution $\underline{\sigma}^D = \bar{\sigma}^D = \hat{\sigma}^D$ involving no pooling region. We can then state the following proposition that characterizes the equilibrium strategy.

Proposition 3 *If $\hat{\sigma}^D \leq \tilde{\sigma}$, there exists a unique separating symmetric Nash equilibrium. The strictly increasing bidding equilibrium strategies read*

$$P^D(s) = \bar{P}_0(1 - K(\hat{\sigma}^D | s)) + \int_s^{\hat{\sigma}^D} p(x, x) dK(x | s) \quad \forall s \leq \hat{\sigma}^D. \quad (20)$$

¹³This is checked in the proof of Lemma 4.

If $\hat{\sigma}^D > \tilde{\sigma}$, there exists a unique symmetric Nash equilibrium in increasing bidding equilibrium strategies with partial pooling where

$$P^D(s) = \begin{cases} \bar{P}_0(1 - K(\underline{\sigma}^D|s)) + \int_s^{\underline{\sigma}^D} p(x, x)dK(x|s) & \text{for } s \leq \underline{\sigma}^D \\ \bar{P}_0 & \text{for } \underline{\sigma}^D < s \leq \bar{\sigma}^D \end{cases} \quad (21a)$$

$$(21b)$$

where $K(x|s) = 1 - \exp\left(-\int_s^x \frac{\kappa g(\tau|\tau)}{1 - \kappa G(\tau|\tau)} d\tau\right)$ is an increasing function with $K(s|s) = 0$ and $K(1|s) \leq 1$.

4.2 Impact of the strength of competition

Contrary to the pool, the region in which insurance companies submit bids now depends on the competition strength. The thresholds matter not only for the leader's but also for the follower's strategy so that the leader and the follower capacities (and thus the strength of competition) are important to determine the bidding regions strategies. The higher κ , the lower the capacity is left to the follower. As a consequence, the three thresholds $\hat{\sigma}^D$, $\underline{\sigma}^D$ and $\bar{\sigma}^D$ depend on κ .

Lemma 5 *In the separating equilibrium, the signal limiting the bidding region is decreasing in κ*

$$\frac{\partial \hat{\sigma}^D}{\partial \kappa} \leq 0.$$

In the semi-pooling equilibrium, the lowest (resp. highest) bound of the pooling region is increasing (resp. decreasing) in κ

$$\frac{\partial \underline{\sigma}^D}{\partial \kappa} \geq 0 \text{ and } \frac{\partial \bar{\sigma}^D}{\partial \kappa} \leq 0.$$

A direct consequence of this lemma can be stated in the following proposition.

Proposition 4 *If $\bar{P}_0 \leq \mathbb{E}[p(S_i, S_j)|S_j = \tilde{\sigma}]$, then the separating equilibrium exists $\forall \kappa \in [0, 1]$.*

If $\bar{P}_0 > \mathbb{E}[p(S_i, S_j)|S_j = \tilde{\sigma}]$ then there exists a unique κ^ such that the separating equilibrium (resp. the semi-pooling equilibrium) exists if and only if $\kappa \geq \kappa^*$ (resp. $\kappa < \kappa^*$).*

When $\bar{P}_0 \leq \mathbb{E}[p(S_i, S_j)|S_j = \tilde{\sigma}]$ the reserve price is too low with respect to the expected actuarial premium, so that bidders refrain from insuring large risks and $\hat{\sigma}^D < \tilde{\sigma}$.

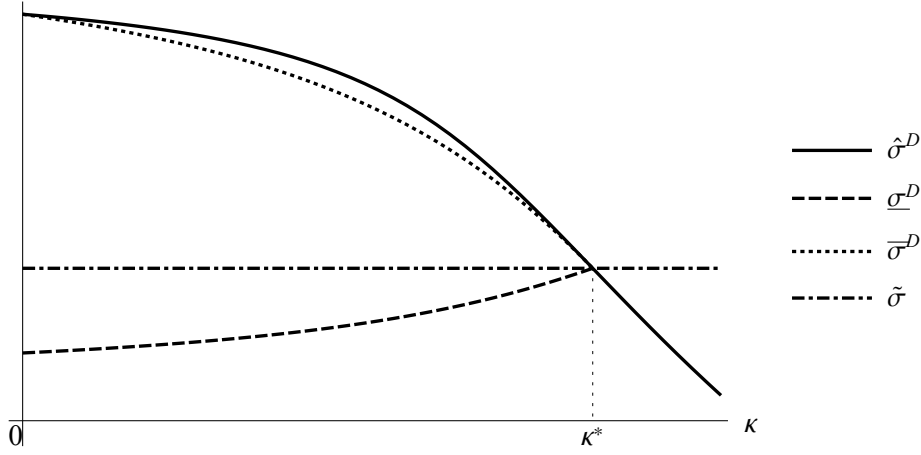


Figure 4: The different thresholds as a function of κ .

On the contrary, when $\bar{P}_0 \leq \mathbb{E}[p(S_i, S_j)|S_j = \tilde{\sigma}]$ (as illustrated in Figure 4), the semi-pooling equilibrium exists when $\kappa \leq \kappa^*$ and the separating equilibrium exists when $\kappa \geq \kappa^*$. In the semi pooling equilibrium, the outcome of the pooling region $[\underline{\sigma}^D, \bar{\sigma}^D]$ is the monopoly outcome (\bar{P}_0). The lower the competition, the larger this region. When competition becomes too intense, this region disappears and the separating equilibrium exists. In this case, the capacity left to the follower decreases so that bidders refrain from taking high risks ($\hat{\sigma}^D$ decreases). There is then a non-monotony in the signal that generates the monopoly premium.

Corollary 1 *The regions in which the capacity is fully served is decreasing with competition.*

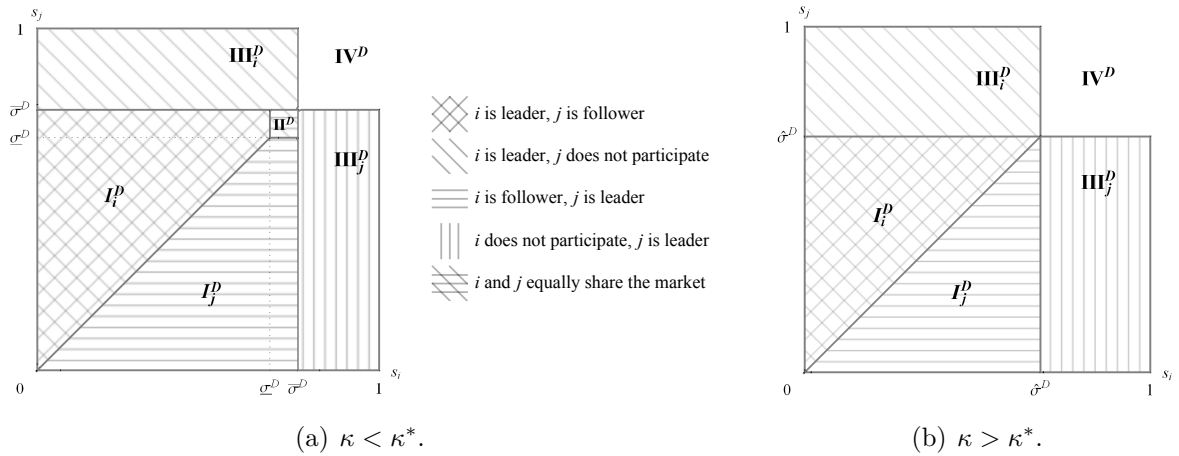


Figure 5: Regions in the two equilibria.

Contrary to the pool, the two insurers obtain different premiums. When there is full coverage, the comparative statics with respect to κ is not straightforward for the premium P^D . Indeed, even if the direct effect that tends to decrease the premium level is the same, there exists an indirect effect that comes from the fact that the threshold $\min(\hat{\sigma}^D, \underline{\sigma}^D)$ depends on κ . This indirect effect writes:

$$-\frac{\partial \min(\hat{\sigma}^D, \underline{\sigma}^D)}{\partial \kappa} \left(\bar{P}_0 - p(\min(\hat{\sigma}^D, \underline{\sigma}^D), \min(\hat{\sigma}^D, \underline{\sigma}^D)) \right) \frac{dK(x|s)}{dx} \Big|_{x=\min(\hat{\sigma}^D, \underline{\sigma}^D)}.$$

If $\min(\hat{\sigma}^D, \underline{\sigma}^D) = \underline{\sigma}^D$ (when the semi-pooling equilibrium exists, $\kappa < \kappa^*$), the indirect effect is negative as the direct effect. In this case, P^D decreases when competition increases until κ^* . At the opposite, the indirect effect is positive in the separating equilibrium since $\hat{\sigma}^D$ is a decreasing function of κ . As we can see in Figure 6(a), the total effect is ambiguous. Also, the variation in premium is ambiguous when κ increases so that the equilibrium switches from semi-pooling to separating as we can see in Figure 6(b).

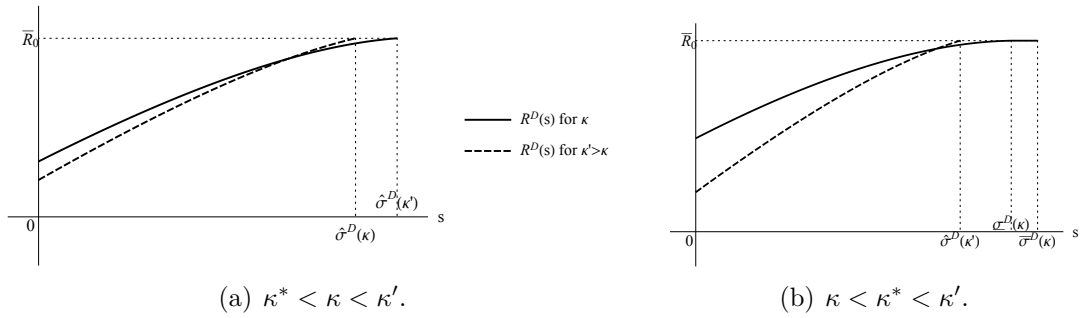


Figure 6: Premiums in the two equilibria.

4.3 Discriminatory auction with reentry

We now introduce the possibility of reentry. A firm that refrains from bidding in the first round has the opportunity to enter the syndicate in the second round at the leader premium. As in the case without reentry, the equilibrium may be separating or semi-pooling. Reentry affects the various thresholds modifying the cases where equilibrium is separating or semi-pooling. Note that the shape of the premium remains the same.

Separating equilibrium. The threshold $\hat{\sigma}_r^D$ is determined by the incentive compatibility constraint.¹⁴

¹⁴This threshold is the maximal signal for which an insurer bids in the first round.

$$\begin{aligned}
& \int_{\hat{\sigma}_r^D}^1 (\bar{P}_0 - p(\hat{\sigma}_r^D, s_j)) g(s_j | \hat{\sigma}_r^D) ds_j + \int_0^{\hat{\sigma}_r^D} (1 - \kappa) (\bar{P}_0 - p(\hat{\sigma}_r^D, s_j)) g(s_j | \hat{\sigma}_r^D) ds_j \\
&= \int_0^{\hat{\sigma}_r^D} (1 - \kappa) (P_r^D(s_j) - p(\hat{\sigma}_r^D, s_j))_+ g(s_j | \hat{\sigma}_r^D) ds_j.
\end{aligned} \tag{22}$$

The only difference with Equation (16) comes from the possibility for the follower to enter in the second round at the leader's premium, $P_r^D(s_j)$.

Lemma 6 *Reentry yields to more conservative strategies ($\hat{\sigma}_r^D \leq \hat{\sigma}^D$). Moreover, when $\hat{\sigma}^D < \tilde{\sigma}$, insurers bid less aggressively ($P_r^D(s) > P^D(s)$, $\forall s \leq \hat{\sigma}_r^D$).*

Separating equilibria are therefore more frequent when reentry is allowed.

Semi-pooling equilibrium. When $\hat{\sigma}_r^D > \tilde{\sigma}$, the equilibrium involves some pooling on a region $[\underline{\sigma}_r^D, \bar{\sigma}_r^D]$. Again, the difference with the case where reentry is not allowed comes from the possible reentry when an insurer observed a signal greater than $\bar{\sigma}_r^D$ so that the incentive compatibility constraints write

$$\left\{ \begin{aligned}
& \int_{\underline{\sigma}_r^D}^{\bar{\sigma}_r^D} (\bar{P}_0 - p(\underline{\sigma}_r^D, s_j)) g(s_j | \underline{\sigma}_r^D) ds_j = 0 & (23a) \\
& \int_{\bar{\sigma}_r^D}^1 (\bar{P}_0 - p(\bar{\sigma}_r^D, s_j)) g(s_j | \bar{\sigma}_r^D) ds_j + \int_{\underline{\sigma}_r^D}^{\bar{\sigma}_r^D} \left(1 - \frac{\kappa}{2}\right) (\bar{P}_0 - p(\bar{\sigma}_r^D, s_j)) g(s_j | \bar{\sigma}_r^D) ds_j \\
& + \int_0^{\underline{\sigma}_r^D} (1 - \kappa) (\bar{P}_0 - p(\bar{\sigma}_r^D, s_j)) g(s_j | \bar{\sigma}_r^D) ds_j = \int_{\underline{\sigma}_r^D}^{\bar{\sigma}_r^D} (1 - \kappa) (\bar{P}_0 - p(\bar{\sigma}_r^D, s_j))_+ g(s_j | \bar{\sigma}_r^D) ds_j \\
& + \int_0^{\underline{\sigma}_r^D} (1 - \kappa) (P_r^D(s_j) - p(\bar{\sigma}_r^D, s_j))_+ g(s_j | \bar{\sigma}_r^D) ds_j. & (23b)
\end{aligned} \right.$$

The comparative statics with respect to κ , and thus whether the equilibrium is separating or semi-pooling, is more involved since the endogenous premium P_r^D enters in the definition of the thresholds. However, we can prove that the separating equilibrium always exists in the two extreme cases where $\kappa = 0$ and $\kappa = 1$.

- When $\kappa = 0$, according to Proposition 4, if $P_0 > E[p(S_i, S_j) | S_j = \tilde{\sigma}]$, the no-reentry separating equilibrium does not exist. However, $\hat{\sigma}_r^D = \hat{\sigma}^D < \tilde{\sigma}$ when $\kappa = 0$ so that the re-entry separating equilibrium exists.

- When $\kappa = 1$, we have $\hat{\sigma}_r^D = \hat{\sigma}^P < \hat{\sigma}^D < \tilde{\sigma}$ so that the equilibrium is always separating.

Re-entry separating equilibrium exists for low and large values of κ . The value of the re-entry option decreases with κ since both the premium and the residual capacity decrease. It is therefore less valuable to be the follower and insurers become less conservative. However, when competition becomes more intense, the winner's curse leads insurers to become more conservative. As a consequence, there is a non monotony in the shape of $\hat{\sigma}_r^D$ as Figure 7 shows.

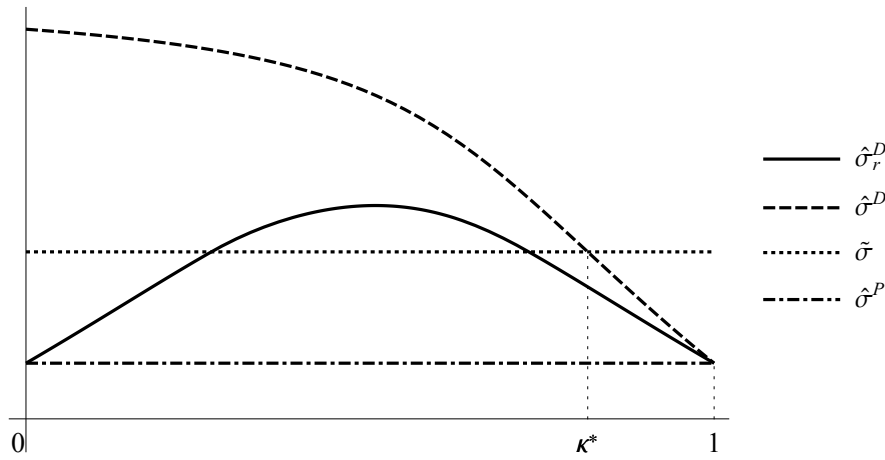


Figure 7: The different thresholds as a function of κ .

5 Discussion

This section discusses the benefit of the pool compared to the standard auctions upon two dimensions, the capacity insured and the premiums. The following lemma compares the thresholds of the different auctions.

Lemma 7 *Insurance companies submit the maximal bid \bar{P}_0 for lower signal's values in the pool auction:*

$$\hat{\sigma}^P \leq \min(\hat{\sigma}_r^D, \underline{\sigma}_r^D) \leq \min(\hat{\sigma}^D, \underline{\sigma}^D).$$

Also, if the separating equilibrium exists, the boundary between regions II_i^P and III_i^P in the pool always lies above $\hat{\sigma}^D$.

Lemma 7 means that regions I^P are always smaller than regions I^D . If both signals are between $\hat{\sigma}^P$ and $\hat{\sigma}^D$, the discriminatory auction offers a full coverage of the risk whereas the pool offers no coverage. If one signal is between $\hat{\sigma}^P$ and $\hat{\sigma}^D$ and the other is larger than $\hat{\sigma}^D$, the pool offers no coverage and the other auctions offer a partial coverage. These two cases happen with a greater probability the stronger the affiliation between the signals. When only one insurer initially bids ($s_j > \hat{\sigma}^D > s_i$), the pool may offer full coverage (in region II_i^P) whereas the other auctions always cover risk only partially. This is more likely when affiliation is weak as summarized in Proposition 5.

Proposition 5 *The strength of affiliation determines which auction format provides the better coverage. Reentry offers more insurance when affiliation is weak.*

We next compare the auctions upon price dimension. It has been already been established that premiums are lower in the uniform auction compared to the discriminatory auction. Also, as we have shown in Lemma 7, we know that the pool premium is larger than both the discriminatory and the uniform premiums for high signals (by continuity around σ^P). It remains to compare the premiums for lower signals. In particular, we would like to establish whether the pool (in which insurers are more “conservative”) yields to aggressive pricing strategy for low signal values.

Lemma 8 *There exist $\sigma^T < \sigma_r^T \in [0, \hat{\sigma}^P]^2$ such that*

- when $s \in [0, \sigma^T]$, $P^P(s) \leq P^D(s) \leq P_r^D(s)$
- when $s \in [\sigma^T, \sigma_r^T]$, $P^D(s) \leq P^P(s) \leq P_r^D(s)$
- when $s \in [\sigma_r^T, \hat{\sigma}^P]$, $P^D(s) \leq P_r^D(s) \leq P^P(s)$

For low signal values, the pool premium is smaller than the premium of both the leader and the follower in the discriminatory auction: the pool is unambiguously better for the insureds. This is affect is stronger when affiliation is weak. Combined with Proposition 5, the pool dominates the two forms of discriminatory auctions. For higher signal values, the leader premium is larger in the pool but the follower premium may be lower. In particular, this effect may be stronger when affiliation is weak.

6 Appendix

6.1 Proof of Lemma 1

We first prove that $\hat{\sigma}^P < \tilde{\sigma}$. Assume by contradiction that $\hat{\sigma}^P \geq \tilde{\sigma}$. Then,

$$\begin{aligned} \int_{\hat{\sigma}^P}^1 (\bar{P}_0 - p(\hat{\sigma}^P, s_j)) g(s_j | \hat{\sigma}^P) ds_j &< (\bar{P}_0 - p(\hat{\sigma}^P, \hat{\sigma}^P)) (1 - G(\hat{\sigma}^P | \hat{\sigma}^P)) \\ &\leq 0 \end{aligned}$$

which contradicts the definition of $\hat{\sigma}^P$.

We introduce functions ψ and \mathcal{L} defined by

$$\psi(x) \equiv \int_x^1 (\bar{P}_0 - p(x, s_j)) g(s_j | x) ds_j \quad (24)$$

$$\mathcal{L}(s|x) \equiv \frac{\frac{dg(s|x)}{dx}}{g(s|x)}. \quad (25)$$

We have $\hat{\sigma}^P$ defined by $\psi(\hat{\sigma}^P) = 0$. Function $s \mapsto \mathcal{L}(s|x)$ is increasing according to Assumption 1.

Let us prove that ψ has a unique zero.

$$\begin{aligned} \psi'(x) &= -(\bar{P}_0 - p(x, x)) g(x|x) - \int_x^1 P_1(x, s_j) g(s_j|x) ds_j \\ &\quad + \int_x^1 (\bar{P}_0 - p(x, s_j)) \mathcal{L}(s_j|x) g(s_j|x) ds_j. \end{aligned}$$

The first two terms are negative (since $\hat{\sigma}^P < \tilde{\sigma}$). Let us focus on the third one

$$\begin{aligned} \int_x^1 (\bar{P}_0 - p(x, s_j)) \mathcal{L}(s_j|x) g(s_j|x) ds_j &= \int_x^{\alpha(x)} (\bar{P}_0 - p(x, s_j)) \mathcal{L}(s_j|x) g(s_j|x) ds_j \\ &\quad + \int_{\alpha(x)}^1 (\bar{P}_0 - p(x, s_j)) \mathcal{L}(s_j|x) g(s_j|x) ds_j \\ &\leq \mathcal{L}(\alpha(x)|x) \int_x^{\alpha(x)} (\bar{P}_0 - p(x, s_j)) g(s_j|x) ds_j \\ &\quad + \mathcal{L}(\alpha(x)|x) \int_{\alpha(x)}^1 (\bar{P}_0 - p(x, s_j)) g(s_j|x) ds_j \\ &= \mathcal{L}(\alpha(x)|x) \psi(x) \end{aligned}$$

where the inequality comes for the fact that \mathcal{L} is increasing in s and that $\bar{P}_0 - p(x, s_j) > 0 \Leftrightarrow s_j < \alpha(x)$. Therefore, the derivative of function ψ is negative when ψ equals zero,

and the zero of function ψ , if it exists is unique. Assumption 2(i) implies that $\psi(0) > 0$. Moreover $\psi(1) = 0$. As a consequence, ψ is positive and then negative as x increases and $\hat{\sigma}^P$ always exists and is unique.

6.2 Proof of Lemma 2

We have that $s_i \mapsto P^P(s_j) - p(s_i, s_j)$ is a decreasing function. We prove the result by showing that $P^P(s_j) - p(\hat{\sigma}^P, s_j) > 0, \forall s_j \leq \hat{\sigma}^P$.

$$\begin{aligned}
P^P(s_j) - p(\hat{\sigma}^P, s_j) &= \bar{P}_0 \left(1 - L(\hat{\sigma}^P|s_j)\right) + \int_{s_j}^{\hat{\sigma}^P} p(x, x) dL(x|s_j) - p(\hat{\sigma}^P, s_j) \\
&= \bar{P}_0 \left(1 - L(\hat{\sigma}^P|s_j)\right) + L(\hat{\sigma}^P|s_j) \int_{s_j}^{\hat{\sigma}^P} p(x, x) \frac{dL(x|s_j)}{L(\hat{\sigma}^P|s_j)} - p(\hat{\sigma}^P, s_j) \\
&\geq \bar{P}_0 \left(1 - L(\hat{\sigma}^P|s_j)\right) + L(\hat{\sigma}^P|s_j) p(\hat{\sigma}^P, s_j) - p(\hat{\sigma}^P, s_j) \\
&= \left(\bar{P}_0 - p(\hat{\sigma}^P, s_j)\right) \left(1 - L(\hat{\sigma}^P|s_j)\right) \\
&> 0
\end{aligned}$$

where the first inequality holds if $\int_{s_j}^{\hat{\sigma}^P} p(x, x) \frac{dL(x|s_j)}{L(\hat{\sigma}^P|s_j)} \geq r(\hat{\sigma}^P, s_j)$ and the second inequality holds since $s_j \leq \hat{\sigma}^P < \tilde{\sigma}$.

The remaining of the proof consists in proving that $\int_{s_j}^{\hat{\sigma}^P} p(x, x) \frac{dL(x|s_j)}{L(\hat{\sigma}^P|s_j)} \geq r(\hat{\sigma}^P, s_j), \forall s_j \leq \hat{\sigma}^P$. As r is symmetric and supermodular (Assumption 2(iii)), it holds that $p(\hat{\sigma}^P, \hat{\sigma}^P) + p(s_j, s_j) \geq 2p(\hat{\sigma}^P, s_j)$ so that it is sufficient to prove that

$$\int_{s_j}^{\hat{\sigma}^P} p(x, x) \frac{dL(x|s_j)}{L(\hat{\sigma}^P|s_j)} \geq \frac{p(s_j, s_j) + p(\hat{\sigma}^P, \hat{\sigma}^P)}{2} \forall s_j \leq \hat{\sigma}^P.$$

An integration by part implies that

$$\int_{s_j}^{\hat{\sigma}^P} p(x, x) \frac{dL(x|s_j)}{L(\hat{\sigma}^P|s_j)} = p(\hat{\sigma}^P, \hat{\sigma}^P) - \int_{s_j}^{\hat{\sigma}^P} \frac{d}{dx} p(x, x) \frac{L(x|s_j)}{L(\hat{\sigma}^P|s_j)} dx.$$

Therefore, inequality (26) reads

$$p(\hat{\sigma}^P, \hat{\sigma}^P) - p(s_j, s_j) - 2 \int_{s_j}^{\hat{\sigma}^P} \frac{d}{dx} p(x, x) \frac{L(x|s_j)}{L(\hat{\sigma}^P|s_j)} dx \geq 0 \forall s_j \leq \hat{\sigma}^P.$$

The derivative of function $s_j \mapsto r(\hat{\sigma}^P, \hat{\sigma}^P) - p(s_j, s_j) - 2 \int_{s_j}^{\hat{\sigma}^P} \frac{d}{dx} p(x, x) \frac{L(x|s_j)}{L(\hat{\sigma}^P|s_j)} dx$ equals $-\frac{d}{dx} p(x, x)|_{x=s_j} < 0$ (since $L(s_j) = 0$). Moreover, as $p(\hat{\sigma}^P, \hat{\sigma}^P) - p(s_j, s_j) - 2 \int_{s_j}^{\hat{\sigma}^P} \frac{d}{dx} p(x, x) \frac{L(x|s_j)}{L(\hat{\sigma}^P|s_j)} dx = 0$ when $s_j = \hat{\sigma}^P$ it follows that $p(\hat{\sigma}^P, \hat{\sigma}^P) - p(s_j, s_j) - 2 \int_{s_j}^{\hat{\sigma}^P} \frac{d}{dx} p(x, x) \frac{L(x|s_j)}{L(\hat{\sigma}^P|s_j)} dx \geq 0 \forall s_j \leq$

$\hat{\sigma}^P$ and the result is proved.

6.3 Proof of Proposition 2

It is straightforward to see that $L(x|s)$ is increasing in κ . Remember that

$$P^P(s) = \bar{P}_0(1 - L(\hat{\sigma}^P|s)) + \int_s^{\hat{\sigma}^P} p(x, x)dL(x|s).$$

As $\hat{\sigma}^P$ is independent from κ , $L(\hat{\sigma}^P|s)$ is increasing in κ as we just underlined. This also implies that $\int_s^{\hat{\sigma}^P} p(x, x)dL(x|s)$ is increasing in κ because, for $\kappa' > \kappa$ function L_κ first order stochastic dominates $L_{\kappa'}$.¹⁵

The boundary between regions II_i and III_i is defined by $\{(s_i, s_j) \in [0, 1]^2 | P^P(s_i) = p(s_i, s_j)\}$. For a given s_j , if κ increases, in order the equality $P^P(s_i) = p(s_i, s_j)$ to hold it must be the case that s_i decreases, do that II_i shrinks. As a consequence, III_i expands.

6.4 Proof of Lemma 3

Introduce function ϕ and φ defined by

$$\phi(x) \equiv \int_0^x (\bar{P}_0 - p(x, s_j)) g(s_j|x) ds_j \quad (26)$$

$$\varphi(x) \equiv \psi(x) + (1 - \kappa)\phi(x) \quad (27)$$

where ψ is defined by equation (24) in the proof of Lemma 1. $\hat{\sigma}^D$ (equation (16)) is defined by $\varphi(\hat{\sigma}^D) = 0$. Observe that the only possibility for such an equality to be satisfied is that $\phi(\hat{\sigma}^D) > 0$ and $\psi(\hat{\sigma}^D) < 0$.

As in the proof of Lemma 1, we are going to prove that φ can cancel only once and that its derivative is negative at this point.

¹⁵The subscript “ κ ” indicates the parameter value, κ .

$$\begin{aligned}
\varphi'(x) &= \psi'(x) + (1 - \kappa)\phi'(x) \\
&= -\left(\bar{P}_0 - p(x, x)\right) g(x|x) - \int_x^1 P_1(x, s_j) g(s_j|x) ds_j \\
&\quad + \int_x^1 \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j + (1 - \kappa) \left(\bar{P}_0 - p(x, x)\right) g(x|x) \\
&\quad - (1 - \kappa) \int_0^x P_1(x, s_j) g(s_j|x) ds_j + (1 - \kappa) \int_0^x \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j \\
&= -\int_x^1 P_1(x, s_j) g(s_j|x) ds_j - (1 - \kappa) \int_0^x P_1(x, s_j) g(s_j|x) ds_j - \kappa \left(\bar{P}_0 - p(x, x)\right) g(x|x) \\
&\quad + \int_x^1 \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j + (1 - \kappa) \int_0^x \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j.
\end{aligned}$$

The first two terms are negative. Let us analyze the last two.

$$\begin{aligned}
&\int_x^1 \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j + (1 - \kappa) \int_x^1 \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j \\
= &\int_x^{\alpha(x)} \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j + \int_{\alpha(x)}^1 \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j \\
&+ (1 - \kappa) \int_x^1 \left(\bar{P}_0 - p(x, s_j)\right) \mathcal{L}(s_j|x) g(s_j|x) ds_j \\
\leq &\mathcal{L}(\alpha(x)|x) \int_x^{\alpha(x)} \left(\bar{P}_0 - p(x, s_j)\right) g(s_j|x) ds_j + \mathcal{L}(\alpha(x)|x) \int_{\alpha(x)}^1 \left(\bar{P}_0 - p(x, s_j)\right) g(s_j|x) ds_j \\
&+ (1 - \kappa) \mathcal{L}(x|x) \int_x^1 \left(\bar{P}_0 - p(x, s_j)\right) g(s_j|x) ds_j \\
\leq &\mathcal{L}(\alpha(x)|x) \int_x^{\alpha(x)} \left(\bar{P}_0 - p(x, s_j)\right) g(s_j|x) ds_j + \mathcal{L}(\alpha(x)|x) \int_{\alpha(x)}^1 \left(\bar{P}_0 - p(x, s_j)\right) g(s_j|x) ds_j \\
&+ (1 - \kappa) \mathcal{L}(\alpha(x)|x) \int_x^1 \left(\bar{P}_0 - p(x, s_j)\right) g(s_j|x) ds_j \\
= &\mathcal{L}(\alpha(x)|x) (\psi(x) + (1 - \kappa)\phi(x)) \\
= &\mathcal{L}(\alpha(x)|x) \varphi(x).
\end{aligned}$$

Therefore the derivative of function φ is negative when φ equals zero, and the zero of function φ , if it exists is unique. Assumption 2(i) implies that $\varphi(0) > 0$ and $\varphi(1) < 0$. This implies that φ is positive and then negative as x increases and that $\hat{\sigma}^D$ always exists and is unique.

6.5 Proof of Lemma 4

We prove first that if the separating equilibrium does not exist ($\tilde{\sigma}^D > \tilde{\sigma}$), then the semi-pooling equilibrium exists and is unique.

The first part of this proof goes through a series of steps. Let us first introduce function I , J and K

$$I(x, y) = \int_x^y (\bar{P}_0 - p(x, t)) g(t|x) dt \quad (28)$$

$$J(x, y) = \int_x^y (\bar{P}_0 - p(y, t)) g(t|y) dt \quad (29)$$

$$H(x, y) = \varphi(y) + \frac{\kappa}{2} J(x, y) \quad (30)$$

where φ is defined by equation (27). Observe that $\psi(x) = I(x, 1)$, $\phi(x) = J(0, x)$ and $\varphi(x) = I(x, 1) + (1 - \kappa)J(0, x)$.

Step 1: We show that $\alpha(\bar{\sigma}^D) \leq \underline{\sigma}^D < \tilde{\sigma} < \alpha(\underline{\sigma}^D) \leq \bar{\sigma}^D \leq \hat{\sigma}^D$.

To prove this step, assume that $(\underline{\sigma}^D, \bar{\sigma}^D)$ is a solution meaning that $I(\underline{\sigma}^D, \bar{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \bar{\sigma}^D) = 0$. $I(\underline{\sigma}^D, \bar{\sigma}^D) = 0$ implies that

$$\begin{aligned} \bar{P}_0 \left(G(\bar{\sigma}^D | \underline{\sigma}^D) - G(\underline{\sigma}^D | \underline{\sigma}^D) \right) &= \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} p(\underline{\sigma}^D, t) g(t | \underline{\sigma}^D) dt \\ &> p(\underline{\sigma}^D, \underline{\sigma}^D) \left(G(\bar{\sigma}^D | \underline{\sigma}^D) - G(\underline{\sigma}^D | \underline{\sigma}^D) \right). \end{aligned}$$

Therefore, $\bar{P}_0 > p(\underline{\sigma}^D, \underline{\sigma}^D)$ implying that $\underline{\sigma}^D < \tilde{\sigma}$.

Remember that $t \mapsto \bar{P}_0 - p(\underline{\sigma}^D, t)$ is a decreasing function. In order to have $I(\underline{\sigma}^D, \bar{\sigma}^D) = 0$, it must be the case that $\bar{P}_0 - p(\underline{\sigma}^D, t)$ is first positive and then negative as t increases from $\underline{\sigma}^D$ to $\bar{\sigma}^D$. In particular, we must have that $\bar{P}_0 - p(\underline{\sigma}^D, \bar{\sigma}^D) < 0$. This implies that $\bar{\sigma}^D > \alpha(\underline{\sigma}^D)$ and $\underline{\sigma}^D > \alpha(\bar{\sigma}^D)$ where function α is defined by equation (5). Note that the symmetry of r and the fact that it is increasing with respect to each of its argument imply that $\alpha = \alpha^{-1}$.

To prove that $\bar{\sigma}^D \leq \hat{\sigma}^D$, we show that $J(\underline{\sigma}^D, \bar{\sigma}^D) < 0$ so that $\varphi(\bar{\sigma}^D) > 0$.

$$\begin{aligned}
J(\underline{\sigma}^D, \bar{\sigma}^D) &= \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(t, \bar{\sigma}^D)) g(t|\underline{\sigma}^D) \frac{g(t|\bar{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\
&\leq \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(t, \underline{\sigma}^D)) g(t|\underline{\sigma}^D) \frac{g(t|\bar{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\
&= \int_{\underline{\sigma}^D}^{\alpha(\underline{\sigma}^D)} (\bar{P}_0 - p(t, \underline{\sigma}^D)) g(t|\underline{\sigma}^D) \frac{g(t|\bar{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt + \int_{\alpha(\underline{\sigma}^D)}^{\bar{\sigma}^D} (\bar{P}_0 - p(t, \underline{\sigma}^D)) g(t|\underline{\sigma}^D) \frac{g(t|\bar{\sigma}^D)}{g(t|\underline{\sigma}^D)} dt \\
&\leq \frac{g(\alpha(\underline{\sigma}^D)|\bar{\sigma}^D)}{g(\alpha(\underline{\sigma}^D)|\underline{\sigma}^D)} I(\underline{\sigma}^D, \bar{\sigma}^D) \\
&= 0.
\end{aligned}$$

The second inequality holds because $t \mapsto \frac{g(t|y)}{g(t|x)}$ is an increasing function $\forall x \leq y$.

It therefore holds that $\alpha(\bar{\sigma}^D) \leq \underline{\sigma}^D \leq \tilde{\sigma} \leq \alpha(\underline{\sigma}^D) \leq \bar{\sigma}^D \leq \hat{\sigma}^D$. This allows us to define the region $\mathcal{D} \equiv \{(x, y) \in [0, 1]^2 | \alpha(y) \leq x \leq \tilde{\sigma} \leq \alpha(x) \leq y \leq \hat{\sigma}^D\}$ to which the solution to the following system should belong to

$$\begin{cases} I(x, y) &= 0 \\ H(x, y) &= 0. \end{cases}$$

Step 2: We show that $x_I(y)$ defined by $I(x_I(y), y) = 0$ on $\mathcal{D}_y = \{\tilde{\sigma} \leq y \leq \hat{\sigma}^D | \alpha(y) \leq x_I(y)\}$ is a decreasing function.

The implicit function theorem implies that

$$\frac{dx_I(y)}{dy} = -\frac{I_1(x_I(y), y)}{I_2(x_I(y), y)}.$$

$I_2(x_I(y), y) = (\bar{P}_0 - p(x_I(y), y))g(y|x_I(y)) \leq 0$ since $\alpha(y) \leq x_I(y)$ (or equivalently $y \geq \alpha(x_I(y))$).

$$\begin{aligned}
I_1(x_I(y), y) &= -(\bar{P}_0 - p(x_I(y), x_I(y)))g(x_I(y)|x_I(y)) - \int_{x_I(y)}^y P_1(x_I(y), t)g(t|x_I(y)) \\
&\quad + \int_{x_I(y)}^y (\bar{P}_0 - p(x_I(y), t))\mathcal{L}(t|x_I(y))g(t|x_I(y))dt
\end{aligned}$$

The first two terms are negative (the first because $x_I(y) \leq \tilde{\sigma}$). As for the third term, using the fact that $t \mapsto \mathcal{L}(t|x_I(y))$ is an increasing function and as we did in the proofs

of Lemmas 1 and 3

$$\begin{aligned}
& \int_{x_I(y)}^y (\bar{P}_0 - p(x_I(y), t)) \mathcal{L}(t|x_I(y)) g(t|x_I(y)) dt \\
& \leq \mathcal{L}(\alpha(x_I(y))|x_I(y)) \int_{x_I(y)}^y (\bar{P}_0 - p(x_I(y), t)) g(t|x_I(y)) dt \\
& = 0.
\end{aligned}$$

This implies that $y \mapsto x_I(y)$ is a decreasing function.

Note moreover that $x_I(\tilde{\sigma}) = \tilde{\sigma}$ and that $x_I(\hat{\sigma}^D) > \alpha(\hat{\sigma}^D)$. To prove this last inequality, assume by contradiction that $x_I(\hat{\sigma}^D) \leq \alpha(\hat{\sigma}^D)$. Knowing that $I(x_I(y), y) = 0$ and since $p(x_I(\hat{\sigma}^D), t) < p(\alpha(\hat{\sigma}^D), t)$ this implies that

$$\int_{x_I(\hat{\sigma}^D)}^{\hat{\sigma}^D} (\bar{P}_0 - p(\alpha(\hat{\sigma}^D), t)) g(t|x_I(\hat{\sigma}^D)) dt < 0.$$

However, as $p(\alpha(\hat{\sigma}^D), t) < p(\alpha(\hat{\sigma}^D), \hat{\sigma}^D) = \bar{P}_0$,

$$\int_{x_I(\hat{\sigma}^D)}^{\hat{\sigma}^D} (\bar{P}_0 - p(\alpha(\hat{\sigma}^D), t)) g(t|x_I(\hat{\sigma}^D)) dt > 0,$$

hence a contradiction implying that $x_I(\hat{\sigma}^D) > \alpha(\hat{\sigma}^D)$.

The last property that remains to be showed for this function x_I is that $y \mapsto x_I(y)$ and $y \mapsto \alpha(y)$ only cross once when $y \in [\tilde{\sigma}, \hat{\sigma}^D]$. This is not a priori obvious since the two functions are decreasing. We know that $x_I(\tilde{\sigma}) = \alpha(\tilde{\sigma}) = \tilde{\sigma}$. Assume that there exists $\bar{y} \in (\tilde{\sigma}, \hat{\sigma}^D]$ such that $x_I(\bar{y}) = \alpha(\bar{y})$. By definition of x_I , this implies that

$$\int_{\alpha(\bar{y})}^{\bar{y}} (\bar{P}_0 - p(\alpha(\bar{y}), t)) g(t|\alpha(\bar{y})) dt = 0.$$

However, $p(\alpha(\bar{y}), t) < p(\alpha(\bar{y}), \bar{y}) = \bar{P}_0$, $\forall t \in [\alpha(\bar{y}), \bar{y}]$, so that it is not possible that the integral equals 0. Therefore such an \bar{y} does not exist. As a consequence, $y \mapsto x_I(y)$ and $y \mapsto \alpha(y)$ only cross for $y = \tilde{\sigma}$, and $\forall y \in [\tilde{\sigma}, \hat{\sigma}^D]$, $x_I(y) > \alpha(y)$.

Step 3: we show that $y_H(x)$ defined by $H(x, y_H(x)) = 0$ on $\mathcal{D}_x = \{\alpha(\hat{\sigma}^D) \leq x \leq \tilde{\sigma} | \alpha(x) \leq y_H(x)\}$ is an increasing function.

The implicit function theorem implies that

$$\frac{dy_H(x)}{dx} = -\frac{H_1(x, y_H(x))}{H_2(x, y_H(x))}.$$

Remembering that $H(x, y) = \varphi(y) + (\kappa/2)J(x, y)$ (where φ is defined by equation (27)), this reads

$$\frac{dy_H(x)}{dx} = -\frac{\frac{\kappa}{2}J_1(x, y_H(x))}{\varphi'(y_H(x)) + \frac{\kappa}{2}J_2(x, y_H(x))}.$$

$$\begin{aligned} J_1(x, y_H(x)) &= -(\bar{P}_0 - p(x, y_H(x)))g(x|y_H(x)) \\ &> -(\bar{P}_0 - p(x, \alpha(x)))g(x|y_H(x)) \\ &= 0. \end{aligned}$$

$$\begin{aligned} H_2(x, y_H(x)) &= -(\bar{P}_0 - p(y_H(x), y_H(x)))g(y_H(x)|y_H(x)) - \int_{y_H(x)}^1 P_1(y_H(x), t)g(t|y_H(x))dt \\ &\quad + \int_{y_H(x)}^1 (\bar{P}_0 - p(y_H(x), t)g(t|y_H(x)))\mathcal{L}(t|y_H(x))dt \\ &\quad + (1 - \kappa) \left((\bar{P}_0 - p(y_H(x), y_H(x)))g(y_H(x)|y_H(x)) - \int_0^{y_H(x)} P_2(t, y_H(x))g(t, |y_H(x))dt \right. \\ &\quad \left. + \int_0^{y_H(x)} (\bar{P}_0 - p(t, y_H(x)))g(t|y_H(x))\mathcal{L}(t|y_H(x))dt \right) \\ &\quad + \frac{\kappa}{2} \left((\bar{P}_0 - p(y_H(x), y_H(x)))g(y_H(x), y_H(x)) - \int_x^{y_H(x)} P_2(t, y_H(x))g(t|y_H(x))dt \right. \\ &\quad \left. + \int_x^{y_H(x)} (\bar{P}_0 - p(t, y_H(x)))g(t|y_H(x))\mathcal{L}(t|y_H(x))dt \right) \\ &= -\frac{\kappa}{2}(\bar{P}_0 - p(y_H(x), y_H(x)))g(y_H(x)|y_H(x)) - \int_{y_H(x)}^1 P_1(y_H(x), t)g(t|y_H(x))dt \\ &\quad - (1 - \kappa) \int_0^{y_H(x)} P_2(t, y_H(x))g(t, |y_H(x))dt - \frac{\kappa}{2} \int_x^{y_H(x)} P_2(t, y_H(x))g(t|y_H(x))dt \\ &\quad + \int_{y_H(x)}^1 (\bar{P}_0 - p(y_H(x), t)g(t|y_H(x)))\mathcal{L}(t|y_H(x))dt \\ &\quad + (1 - \kappa) \int_0^{y_H(x)} (\bar{P}_0 - p(t, y_H(x)))g(t|y_H(x))\mathcal{L}(t|y_H(x))dt \\ &\quad + \frac{\kappa}{2} \int_x^{y_H(x)} (\bar{P}_0 - p(t, y_H(x)))g(t|y_H(x))\mathcal{L}(t|y_H(x))dt. \end{aligned}$$

The first four terms are negative. Let us analyze the last three terms.

$$\begin{aligned}
& \int_{y_H(x)}^1 (\bar{P}_0 - p(y_H(x), t)) g(t|y_H(x)) \mathcal{L}(t|y_H(x)) dt \\
& + (1 - \kappa) \int_0^{y_H(x)} (\bar{P}_0 - p(t, y_H(x))) g(t|y_H(x)) \mathcal{L}(t|y_H(x)) dt \\
& + \frac{\kappa}{2} \int_x^{y_H(x)} (\bar{P}_0 - p(t, y_H(x))) g(t|y_H(x)) \mathcal{L}(t|y_H(x)) dt \\
\leq & \mathcal{L}(y_H(x)|y_H(x)) I(y_H(x), 1) + (1 - \kappa) \mathcal{L}(\alpha(y_H(x))|y_H(x)) J(0, y_H(x)) + \frac{\kappa}{2} \mathcal{L}(x|y_H(x)) J(0, y_H(x)) \\
\leq & \mathcal{L}(\alpha(y_H(x))|y_H(x)) \left(I(y_H(x), 1) + (1 - \kappa) J(0, y_H(x)) + \frac{\kappa}{2} J(0, y_H(x)) \right) \\
= & \mathcal{L}(\alpha(y_H(x))|y_H(x)) H(x, y_H(x)).
\end{aligned}$$

The first inequality holds because

- the first integral is negative. Indeed, $\tilde{\sigma} \leq y_H(x) \leq t$ so that $\bar{P}_0 - p(y_H(x), t) \leq 0$, $\forall t \in [y_H(x), 1]$;
- the second integral is positive (resp. negative) when $t \in [0, \alpha(y_H(x))]$ (resp. $t \in [\alpha(y_H(x)), y_H(x)]$);
- the third integral is negative. Indeed, $\alpha(y_H(x)) \leq x \leq t$ so that $\bar{P}_0 - p(y_H(x), t) \leq 0$, $\forall t \in [x, y_H(x)]$;
- function $t \mapsto \mathcal{L}(t|y_H(x))$ is increasing (Assumption 1).

The second inequality holds because $\alpha(y_H(x)) \leq x \leq y_H(x)$ and because function $t \mapsto \mathcal{L}(t|y_H(x))$ is increasing. This implies that $H_2(x, y_H(x)) \leq 0$ and therefore y_H is increasing function.

Moreover note that $y_H(\tilde{\sigma}) \in [\tilde{\sigma}, \hat{\sigma}^D]$. Suppose by contradiction that $y_H(\tilde{\sigma}) > \hat{\sigma}^D$. In this case, $\varphi(y_H(\tilde{\sigma})) < 0$ and $J(\tilde{\sigma}, y_H(\tilde{\sigma})) > 0$. This implies that

$$\begin{aligned}
0 & < \int_{\tilde{\sigma}}^{y_H(\tilde{\sigma})} (\bar{P}_0 - p(y_H(\tilde{\sigma}), t)) g(t|y_H(\tilde{\sigma})) dt \\
& < \int_{\tilde{\sigma}}^{y_H(\tilde{\sigma})} (\bar{P}_0 - p(\tilde{\sigma}, t)) g(t|y_H(\tilde{\sigma})) dt \\
& < 0,
\end{aligned}$$

leading to a contradiction. Therefore $y_H(\tilde{\sigma}) \leq \hat{\sigma}^D$. The same reasoning implies that $y_H(\tilde{\sigma}) \leq \tilde{\sigma}$.

Step 4: We show that the solution $(\underline{\sigma}^D, \bar{\sigma}^D)$ is unique.

x_I is a decreasing function such that $x_I(\tilde{\sigma}) = \tilde{\sigma}$, $x_I(\hat{\sigma}^D) = \tilde{\sigma}$ and Y_H is an increasing function such that $y_H(\tilde{\sigma}) \in [\tilde{\sigma}, \hat{\sigma}^D]$. As $x_I(y) > \alpha(y)$, $\forall y \in (\tilde{\sigma}, \hat{\sigma}^D]$, the two function cross only once on \mathcal{D} . This intersection point that is unique corresponds to the unique solution of the system that is $(\underline{\sigma}^D, \bar{\sigma}^D)$.

The last part of the proof consists in showing that if the semi-pooling equilibrium exists, then the separating equilibrium does not exist. We are going to prove that if the separating equilibrium exists then the semi-pooling equilibrium does not exist. Assume therefore that $\hat{\sigma}^P < \tilde{\sigma}$. We have proven in Step 1 that $\bar{\sigma}^P \leq \hat{\sigma}^P$, this implies that $\underline{\sigma}^D < \bar{\sigma}^D < \tilde{\sigma}$. But in this case, $I(\underline{\sigma}^D, \bar{\sigma}^D) > 0$. Therefore, if $\hat{\sigma}^P < \tilde{\sigma}$, there do not exist $(\underline{\sigma}^D, \bar{\sigma}^D)$ satisfying $I(\underline{\sigma}^D, \bar{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \bar{\sigma}^D) = 0$.

It remains to prove that

$$\int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(s, t)) g(t|s) dt \leq 0 \forall s \in [\underline{\sigma}^D, \bar{\sigma}^D].$$

$$\begin{aligned} \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(s, t)) g(t|s) dt &= \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(t, s)) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\ &\leq \int_{\underline{\sigma}^D}^{\bar{\sigma}^D} (\bar{P}_0 - p(t, \underline{\sigma}^D)) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\ &= \int_{\underline{\sigma}^D}^{\alpha(\underline{\sigma}^D)} (\bar{P}_0 - p(t, \underline{\sigma}^D)) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\ &\quad + \int_{\alpha(\underline{\sigma}^D)}^{\bar{\sigma}^D} (\bar{P}_0 - p(t, \underline{\sigma}^D)) g(t|\underline{\sigma}^D) \frac{g(t|s)}{g(t|\underline{\sigma}^D)} dt \\ &\leq \frac{g(\alpha(\underline{\sigma}^D)|s)}{g(\alpha(\underline{\sigma}^D)|\underline{\sigma}^D)} I(\underline{\sigma}^D, \bar{\sigma}^D) \\ &= 0. \end{aligned}$$

6.6 Proof of Lemma 5

$\varphi(\hat{\sigma}^D) = \psi(\hat{\sigma}^D) + (1 - \kappa) \phi(\hat{\sigma}^D) = 0$. As we noted in the proof of Lemma 3, $\phi(\hat{\sigma}^D) > 0$.

The implicit function theorem implies that

$$\frac{\partial \hat{\sigma}^D}{\partial \kappa} = \frac{\phi(\hat{\sigma}^D)}{\psi'(\hat{\sigma}^D) + (1 - \kappa) \phi'(\hat{\sigma}^D)} = \frac{\phi(\hat{\sigma}^D)}{\varphi'(\hat{\sigma}^D)}.$$

In Lemma 3, we have proved that $\varphi'(\hat{\sigma}^D) \leq 0$ so that $\frac{\partial \hat{\sigma}^D}{\partial \kappa} \leq 0$.

If $\hat{\sigma}^D > \tilde{\sigma}$, $\underline{\sigma}^D$ and $\bar{\sigma}^D$ are such that

$$\begin{cases} I(\underline{\sigma}^D, \bar{\sigma}^D) = 0 \\ H(\underline{\sigma}^D, \bar{\sigma}^D) = 0. \end{cases}$$

The implicit function theorem implies that

$$\begin{aligned} \frac{\partial \bar{\sigma}^D}{\partial \kappa} &= \frac{J(0, \bar{\sigma}^D) - \frac{1}{2}J(\underline{\sigma}^D, \bar{\sigma}^D)}{I_1(\bar{\sigma}^D, 1) + (1 - \kappa)J_1(0, \bar{\sigma}^D) + \frac{\kappa}{2}J_2(\underline{\sigma}^D, \bar{\sigma}^D) - \frac{\kappa}{2}J_1(\underline{\sigma}^D, \bar{\sigma}^D) \frac{I_2(\underline{\sigma}^D, \bar{\sigma}^D)}{I_1(\underline{\sigma}^D, \bar{\sigma}^D)}} \\ \frac{\partial \underline{\sigma}^D}{\partial \kappa} &= -\frac{\partial \bar{\sigma}^D}{\partial \kappa} \frac{I_2(\underline{\sigma}^D, \bar{\sigma}^D)}{I_1(\underline{\sigma}^D, \bar{\sigma}^D)}. \end{aligned}$$

We have proven in Step 3 of the proof of Lemma 4 that $H_2(\underline{\sigma}^D, \bar{\sigma}^D) = I_1(\bar{\sigma}^D, 1) + (1 - \kappa)J_1(0, \bar{\sigma}^D) + \frac{\kappa}{2}J_2(\underline{\sigma}^D, \bar{\sigma}^D) \leq 0$ and that $J_1(\underline{\sigma}^D, \bar{\sigma}^D) \geq 0$. In Step 2 of the proof of Lemma 4, we also showed that $\frac{I_1(\underline{\sigma}^D, \bar{\sigma}^D)}{I_2(\underline{\sigma}^D, \bar{\sigma}^D)} \geq 0$. This implies that the denominator of $\frac{\partial \bar{\sigma}^D}{\partial \kappa}$ is negative and that $\frac{\partial \bar{\sigma}^D}{\partial \kappa}$ and $\frac{\partial \underline{\sigma}^D}{\partial \kappa}$ have opposite signs.

$$\begin{aligned} J(0, \bar{\sigma}^D) - \frac{1}{2}J(\underline{\sigma}^D, \bar{\sigma}^D) &= \frac{1}{\kappa} \left(I(\bar{\sigma}^D, 1) + J(0, \bar{\sigma}^D) \right) \\ &= \frac{1}{\kappa} \left(I(\bar{\sigma}^D, 1) + (1 - \kappa)J(0, \bar{\sigma}^D) + \kappa J(0, \bar{\sigma}^D) \right). \end{aligned}$$

In Step 1 of the proof of Lemma 4, we also proved that $J(\underline{\sigma}^D, \bar{\sigma}^D) \leq 0$. Together with $H(\underline{\sigma}^D, \bar{\sigma}^D) = 0$ implies that

$$I(\bar{\sigma}^D, 1) + (1 - \kappa)J(0, \bar{\sigma}^D) = -\frac{\kappa}{2}J(\underline{\sigma}^D, \bar{\sigma}^D) \geq 0.$$

Moreover, as $I(\bar{\sigma}^D, 1) \leq 0$ and $\kappa \in (0, 1)$, it must be the case that $J(0, \bar{\sigma}^D) \geq 0$. It follows that $J(0, \bar{\sigma}^D) - \frac{1}{2}J(\underline{\sigma}^D, \bar{\sigma}^D) \geq 0$. As a consequence, $\frac{\partial \bar{\sigma}^D}{\partial \kappa} \leq 0$ and $\frac{\partial \underline{\sigma}^D}{\partial \kappa} \geq 0$.

6.7 Proof of Proposition 4

Remember that the separating equilibrium exists if and only if $\hat{\sigma}^D < \tilde{\sigma}$. Let us analyze the function $\kappa \mapsto \hat{\sigma}^D - \tilde{\sigma}$. Thanks to Lemma 5, we know that this function is decreasing ($\tilde{\sigma}$ is independent of κ). When $\kappa = 1$, observe that $\hat{\sigma}^D = \hat{\sigma}^P$, so that $\hat{\sigma}^D - \tilde{\sigma} = \hat{\sigma}^P - \tilde{\sigma} \leq 0$. If, when $\kappa = 0$, $\hat{\sigma}^P - \tilde{\sigma} < 0$, then $\forall \kappa \in [0, 1]$, $\hat{\sigma}^D < \tilde{\sigma}$ and the separating equilibrium always exists. If, on the contrary, when $\kappa = 0$, $\hat{\sigma}^P - \tilde{\sigma} > 0$, then there exists a unique κ^*

such that the separating equilibrium (resp. semi-pooling equilibrium) exists if and only if $\kappa \geq \kappa^*$ (resp. $\kappa < \kappa^*$). The rest of the proof consists in proving that the comparison of $\hat{\sigma}^P$ to $\tilde{\sigma}$ when $\kappa = 0$ comes down to comparing \bar{P}_0 to $\int_0^1 p(\tilde{\sigma}, s)g(s|\tilde{\sigma})ds = \mathbb{E}[p(S_i, S_j)|S_j = \tilde{\sigma}]$.

When $\kappa = 0$, $\hat{\sigma}^D$ is implicitly defined by

$$\int_0^1 (\bar{P}_0 - p(\hat{\sigma}^D, t)) g(t|\hat{\sigma}^D)dt = 0.$$

Let us introduce function Λ defined by $\Lambda(x) = \int_0^1 (\bar{P}_0 - p(x, t)) g(t|x)dt$. As in the prof of Lemmas 1 and 3, we can prove that

$$\begin{aligned} \Lambda'(x) &= \int_0^1 -P_1(x, t)g(t|x)dt + \int_0^1 (\bar{P}_0 - p(x, t)) \mathcal{L}(t|x)g(t|x)dt \\ &\leq \int_0^1 -P_1(x, t)g(t|x)dt + \mathcal{L}(\alpha(x)|x)\Lambda(x) \end{aligned}$$

so that $\Lambda'(\hat{\sigma}^D) \leq 0$. Because of Assumption 2(i), $\Lambda(0) > 0$ and $\Lambda(1) < 0$, so that Λ is positive if and only if $x \leq \hat{\sigma}^D$. It follows that $\hat{\sigma}^D > \tilde{\sigma}$ if and only if $\Lambda(\tilde{\sigma}) > 0$ which comes down to the condition stated in the proposition, that is $\bar{P}_0 > \int_0^1 p(\tilde{\sigma}, s)g(s|\tilde{\sigma})ds$.

6.8 Proof of Proposition ??

Separating equilibrium. We first look first for an equilibrium in strictly increasing and symmetric bidding strategy as far as the insurance company wants to participate to the auction. Such a equilibrium is characterized by a threshold $\hat{\sigma}^U$ such that

- when $s_i \leq \hat{\sigma}^U$, firm i bids according to a strictly increasing bidding strategy $P^U(s_i)$ such that $P^U(\hat{\sigma}^U) = \bar{P}_0$,
- when $s_i > \hat{\sigma}^U$, firm i does not participate to the syndicate.

The profit of firm i that observed a signal s_i and bids a risk premium $P^U(b)$ reads

$$\Pi^U(b, s_i) = \begin{cases} \bar{\beta} \int_{\hat{\sigma}^U}^1 (\bar{P}_0 - p(s_i, s_j)) g(s_j|s_i) ds_j \\ \quad + \bar{\beta} \int_b^{\hat{\sigma}^U} (P^U(s_j) - p(s_i, s_j)) g(s_j|s_i) ds_j \\ \quad + (\beta - \bar{\beta}) \int_0^b (P^U(b) - p(s_i, s_j)) g(s_j|s_i) ds_j & \text{for } b \leq \hat{\sigma}^U \quad (31a) \\ 0 & \text{for } b > \hat{\sigma}^U \quad (31b) \end{cases}$$

Let us explain the different terms composing the expression of $\Pi_i^U(b, s_i)$. In this auction format, the risk premium an insurer proposes depends on the signal it observed (or it claimed it observed, b in our case) that determines its position as a leader or a follower, but also on the opponent's signal that determines the risk premium.

- When firm i bids $P^U(b) \leq \bar{P}_0$, three situations may arise:

1. Firm j participates and proposes a risk premium smaller than $P^U(b)$. Firm i turns out to be the syndicate's follower leader and serves the remaining capacity $\beta - \bar{\beta}$ at his proposed price $P^U(b)$;
2. Firm j participates and proposes a risk premium equal to $P^U(s_j)$ greater than $P^U(b)$. Firm i turns out to be the syndicate leader and serves $\bar{\beta}$ at the follower's proposed price $P^U(s_j)$;
3. Firm j does not want to participate to this uniform auction, meaning that it received a signal greater than $\hat{\sigma}^U$. Firm i turns out to be the unique syndicate member and serves $\bar{\beta}$ at its proposed price \bar{P}_0 .

- When firm i observes a signal greater than $\hat{\sigma}^U$, it does not participate to the auction.

Incentive compatibility requires that

$$\int_{\hat{\sigma}^U}^1 (\bar{P}_0 - p(\hat{\sigma}^U, s_j)) g(s_j | \hat{\sigma}^U) ds_j + (1 - \kappa) \int_0^{\hat{\sigma}^U} (\bar{P}_0 - p(\hat{\sigma}^U, s_j)) g(s_j | \hat{\sigma}^U) ds_j = 0. \quad (32)$$

Note that $\hat{\sigma}^U = \hat{\sigma}^D$.

At equilibrium, $P^U(b) = P^U(s_i), \forall s_i \leq \hat{\sigma}^D$ so that

$$\frac{\partial \Pi^U(b, s_i)}{\partial b} \Big|_{b=s_i} = 0, \forall s_i \leq \hat{\sigma}^D.$$

This implies that the equilibrium bid $P^U(s_i)$ satisfies the following differential equation

$$P^{U'}(s_i) = \frac{\kappa}{1 - \kappa} \frac{g(s_i | s_i)}{G(s_i | s_i)} (P^U(s_i) - p(s_i, s_i)). \quad (33)$$

In order the bidding strategy to be strictly increasing, a necessary condition is that

$$P^U(s_i) - p(s_i, s_i) > 0, \forall s_i < \hat{\sigma}^D. \quad (34)$$

As in the discriminatory auction, such a strictly increasing strategy is an equilibrium if and only if $\hat{\sigma}^D \leq \tilde{\sigma}$.

Semi-pooling equilibrium. When $\hat{\sigma}^P > \tilde{\sigma}$, the necessary condition (34), we look for a semi pooling equilibrium such that the equilibrium strategy is characterized by two thresholds $\underline{\sigma}^U$ and $\bar{\sigma}^U$ such that

- when $s_i \in [0, \underline{\sigma}^U]$, firm i bids according to a strictly increasing strategy $P^U(s_i)$,
- when $s_i \in [\underline{\sigma}^U, \bar{\sigma}^U]$, firm i bids \bar{P}_0 ,
- when $s_i > \bar{\sigma}^U$, firm i does not participate to the syndicate.

$\underline{\sigma}^U$ and $\bar{\sigma}^U$ are such that firm i 's profit is continuous at these two values. It follows that

$$\Pi^U(b, s_i) = \begin{cases} \bar{\beta} \int_{\underline{\sigma}^U}^1 \bar{\beta} (\bar{P}_0 - p(s_i, s_j)) g(s_j | s_i) ds_j \\ \quad + \bar{\beta} \int_b^{\underline{\sigma}^U} (P^U(s_j) - p(s_i, s_j)) g(s_j | s_i) ds_j \\ \quad + (\beta - \bar{\beta}) \int_0^b (1 - \kappa) (P^U(b) - p(s_i, s_j)) g(s_j | s_i) ds_j & \text{for } b \leq \underline{\sigma}^U & (35a) \\ \frac{\beta}{2} \int_{\underline{\sigma}^U}^1 (\bar{P}_0 - p(s_i, s_j)) g(s_j | s_i) ds_j \\ \quad + (\beta - \bar{\beta}) \left(\int_0^{\underline{\sigma}^U} (\bar{P}_0 - p(s_i, s_j)) g(s_j | s_i) ds_j \right) & \text{for } \underline{\sigma}^U < b \leq \bar{\sigma}^U & (35b) \\ 0 & \text{for } b > \bar{\sigma}^U & (35c) \end{cases}$$

Contrary to (31), there is an intermediate case that has to be taken into account when firm i bids \bar{P}_0 , that it when firm i bids as if it observed a signal b comprised between $\underline{\sigma}^U$ and $\bar{\sigma}^U$. Three situations may arise.

1. Firm j does not want to participate to this uniform auction, meaning that it received a signal greater than $\bar{\sigma}^U$. Firm i turns out to be the unique syndicate member and serves $\bar{\beta}$ at its proposed price \bar{P}_0 ;
2. Firm j also bids P_0 meaning that firm j also observed a signal comprised between $\underline{\sigma}^U$ and $\bar{\sigma}^U$. Firms i and j therefore share the market and each serves a capacity $\beta/2$ at price \bar{P}_0 ;

3. Firm j bids a risk premium strictly less than P_0 meaning that firm j observed a signal smaller than $\underline{\sigma}^U$. Firm i turns out to be the syndicate's follower leader and serves the remaining capacity $\beta - \bar{\beta}$ at his proposed price \bar{P}_0 .

The values $\underline{\sigma}^U$ and $\bar{\sigma}^U$ are such that

$$\begin{cases} \int_{\underline{\sigma}^U}^{\bar{\sigma}^U} (\bar{P}_0 - p(\underline{\sigma}^U, s_j)) g(s_j | \underline{\sigma}^U) ds_j = 0 & (36a) \\ \int_{\bar{\sigma}^U}^1 (\bar{P}_0 - p(\bar{\sigma}^U, s_j)) g(s_j | \bar{\sigma}^U) ds_j + \int_{\underline{\sigma}^U}^{\bar{\sigma}^U} (1 - \frac{\kappa}{2}) (\bar{P}_0 - p(\bar{\sigma}^U, s_j)) g(s_j | \bar{\sigma}^U) ds_j \\ + \int_0^{\underline{\sigma}^U} (1 - \kappa) (P_0 - p(\bar{\sigma}^U, s_j)) g(s_j | \bar{\sigma}^U) ds_j = 0. & (36b) \end{cases}$$

This system is the same as for the discriminatory auction so that $\underline{\sigma}^U = \underline{\sigma}^D$ and $\bar{\sigma}^U = \bar{\sigma}^D$.

6.9 Proof of Lemma 7

Remember that

- $\hat{\sigma}^P$ is such that $\psi(\hat{\sigma}^P) = 0$ where ψ is defined in Equation (24),
- $\hat{\sigma}^D$ is such that $\varphi(\hat{\sigma}^D) = \psi(\hat{\sigma}^D) + (1 - \kappa)\phi(\hat{\sigma}^D) = 0$ where ϕ and φ are defined in Equations (26) and (27),
- $\underline{\sigma}^D$ and $\bar{\sigma}^D$ are the solution of the system $I(\underline{\sigma}^D, \underline{\sigma}^D) = 0$ and $H(\underline{\sigma}^D, \underline{\sigma}^D) = 0$ where I and J are defined in Equations (29) and (30).

Assume first that $\hat{\sigma}^D \leq \tilde{\sigma}$.

We already noted (see the proof of Lemma 3) that $\psi(\hat{\sigma}^D) < 0$. In addition, we also proved in Lemma 1 that $\psi(x) > 0 \Leftrightarrow x < \hat{\sigma}^P$. As $\psi(\hat{\sigma}^P) = 0$, this implies that $\hat{\sigma}^P \leq \hat{\sigma}^D$.

Assume now that $\hat{\sigma}^D > \tilde{\sigma}$.

$\psi(\underline{\sigma}^D) = I(\underline{\sigma}^D, 1) < I(\underline{\sigma}^D, \underline{\sigma}^D) = 0$. The same reasoning implies that $\underline{\sigma}^P \leq \hat{\sigma}^D$.

6.10 Proof of Proposition 8

To be written.

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