# Financial Equilibrium in a Production Economy without Rational Expectations of Prices: a basic Model of full Existence

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# FINANCIAL EQUILIBRIUM IN A PRODUCTION ECONOMY: WITHOUT RATIONAL EXPECTATIONS OF PRICES: A BASIC MODEL OF FULL EXISTENCE

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# Financial equilibrium in a production economy without rational expectations of prices: a basic model of full existence

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***Abstract***

*We extend our pure-exchange existence of equilibrium theorem, with differential information, private anticipations and no model to forecast prices, to a production economy of all ownership types: sole proprietorship, partnership and corporations. We show that, due to bounded rationality, all agents face a "minimum uncertainty", which typically adds to the 'exogenous uncertainty', on tomorrow's state of nature, an 'endogenous uncertainty' on future spot prices, depending on all agents' private beliefs today. At a sequential equilibrium, any achievable spot price is anticipated as possible by all agents, whose strategies are optimal, ex ante, and market clearing, ex post. We show this equilibrium exists, whenever their anticipations embed the minimum uncertainty set. This result, is stronger than classical ones of generic existence, along Radner (1979) and Hart (1975), and a step towards proving existence in a stochastic production economy without rational expectations of prices.*

**Key words:** sequential equilibrium, temporary equilibrium, perfect foresight, existence, rational expectations, financial markets, asymmetric information, arbitrage.

**JEL Classification**: D52

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# 1 Introduction

This paper extends De Boisdeffre's (20017 b) existence theorem of the basic pure-exchange financial economy with differential information, private anticipations and no model to forecast prices along Radner (1972 & 1979), to a similar economy including companies of all kinds: sole proprietorships, joint ventures, corporations.

The current model has two periods, with an a priori uncertainty upon tomorrow's state of nature, which belongs to a given finite state space, $S$. There are finite sets, $I$, of consumers, and $J$, of producers, and we let $K = I \cup J$ be the set of all agents. Production units, $j \in J$, split in two categories, corporations, $j \in J_1$, whose shares may be exchanged on a stock market, and other companies, $j \in J_2$, of sole proprietors or private partners, which may not. Agents' possible asymmetric information, ex ante, is represented by idiosyncratic private signals, $S_k \subset S$, which correctly inform every agent, $k \in K$, that tomorrow's true state will lie in $S_k$. Thus, the pooled information set, $\underline{\mathbf{S}} := \cap_{k \in K} S_k \neq \varnothing$, is henceforth given, and we let w.l.o.g. $S = \cup_{k \in K} S_k$.

Agents exchange finitely many goods and services on spot markets, $h \in H$, serving as inputs or outputs, to producers, or as final consumption goods, to consumers, and whose prices are privately and idiosyncratically anticipated by every agent. Thus, each agent, $k \in K$, in each state, $s \in S_k$, has a private, typically uncountable, set, $P_s^k \subset P := \{p = \mathbb{R}_{++}^H : \|p\| = 1\}$, of anticipations of the spot prices, which may obtain tomorrow; and we let $\Omega_k := \cup_{s \in S_k} \{s\} \times P_s^k \subset S \times P$ be her anticipation set. Agents, $k \in K$, may also exchange, unrestrictively, finitely many securities, $j \in J_0$, at the first period, whose exogenous payoffs tomorrow are conditional on the state of nature to prevail and may be nominal (i.e., pay in cash) or real (i.e., pay in goods). On financial markets, the typically asymmetric anticipations sets, $(\Omega_k)$, grant no agent an unlimited arbitrage opportunity, non restrictively along De Boisdeffre (2016).

The means and fruits of a production unit, $j \in J$, reward the shareholders of a corporation, if $j \in J_1$, and a sole proprietor or the partners of a joint venture, if $j \in J_2$. Consistently with competition, the company's returns to scale are non increasing. The generic producer maximises the ex ante value of her expected profits, given the observed prices, her anticipation set, and her technology constraints, represented by a production set. Similarly, the consumer, whose preferences are ordered, maximizes the ex ante utility of her consumption plan at market prices, given her budget constraints. A sequential equilibrium obtains when agents optimize these strategies, at clearing prices on all markets, as observed or in the anticipations of all agents.

As a result of their having private characteristics and beliefs, and no forecast function a la Radner (1972 & 1979), agents face an incompressible uncertainty over tomorrow's spot prices, embedded into a so-called "*minimum uncertainty set*". A sequential equilibrium with production is shown to exists, as in the pure exchange economy, if agents' anticipations sets embed that minimum uncertainty set. This result is a step towards proving the existence of equilibrium, in the more general setting of a stochastic production economy, where rational expectations fail. The outline is as follows: Section 2 presents the model, Section 3 states and recalls the proof of the existence Theorem.

# 2 The model

We consider a production economy with two periods, $t \in \{0, 1\}$, and an ex ante uncertainty about which state of nature and which spot price will prevail ex post. Agents have private characteristics and forecasts and exchange goods and services under uncertainty, serving as inputs or for final consumption. They trade assets of all kinds on typically incomplete financial markets. The sets, $I$, $J$, $S$, $H$ and $J_0$, respectively, of consumers, producers, states of nature, goods and services, and assets, are all finite, and we let $K := I \cup J$ be the set of all agents. The non random state at the first period $(t = 0)$ is denoted by $s = 0$ and we let $\Sigma' := \{0\} \cup \Sigma$, for every subset, $\Sigma$, of $S$. Similarly, $l = 0$ denotes the unit of account and we let $H' := \{0\} \cup H$.

### 2.1 Markets and information

Producers and consumers, $k \in K := I \cup J$, exchange goods and services, $h \in H$, on both periods' spot and labour markets, for the purpose of the final consumption of consumers, or the use of inputs by producers, which include raw materials, intermediary goods and labour. We refer to a pair of state and price, $\omega := (s, p_s) \in S \times \mathbb{R}^H$, as a forecast. Producers, $j \in J := J_1 \cup J_2$, are of two types: corporations (when $j \in J_1$), whose shares (called equities) can be exchanged on the stock market, and all other producers, $j \in J_2$, consisting of sole proprietors and joint ventures.

All agents may exchange unrestrictively, at $t = 0$, finitely many assets, or securities, $j \in J_0$ (with $\#J_0 \leqslant \#\underline{\mathbf{S}}$), whose yields, at $t = 1$, are exogenous and conditional on the realization of a forecast, $\omega \in S \times \mathbb{R}^H$. The security price is denoted by $q_0 \in \mathbb{R}^{J_0}$. Consumers may also exchange equities on the stock market, or participations in corporations, $j \in J_1$, whose conditional yields across forecasts are endogenous. The equity price is denoted by $q_1 \in \mathbb{R}^{J_1}$. The generic producer's portfolio set is $\mathbb{R}^{J_0}$, that is, she does not exchange equities. Her portfolio, $z_0 := (z_0^j) \in \mathbb{R}^{J_0}$, summarizes the positions that she may take on each asset, positive, if bought, and negative, if sold short. Assets' exogenous payoffs may be nominal (i.e., pay in cash) or real (i.e., pay in goods, in a subset of $H$)[2] or a mix of both. They define a payoff map,

[2] e.g., assets may not pay off in some service, $h \in H$, such as labour, which is only used as input by producers,

$V : S \times \mathbb{R}^H \to \mathbb{R}^{J_0}$, relating forecasts, $\omega := (s,p) \in S \times \mathbb{R}^H$, to rows, $V(\omega) \in \mathbb{R}^{J_0}$, of all assets' cash payoffs, delivered if state $s$ and price $p$ obtain. The equities' endogenous payoffs will be presented later.

At $t = 0$, each agent, $k \in K$, receives a private information signal, $S_k \subset S$, which correctly informs her that tomorrow's true state will be in $S_k$, and we let $\underline{\mathbf{S}} := \cap_{k\in K} S_k$. Moreover, in each state, $s \in S_k$, the agent has a private set of anticipations of possible spot prices in state $s$, assumed to be a closed subset, $P_s^k$, of $P := \{p \in \mathbb{R}^H_{++} : \|p\| = 1\}$. That is, the agent is only concerned about relative prices. The set of first period prices is restricted to $P_0 := \{(p_0,(q_0,q_1)) \in \mathbb{R}^H_{++} \times \mathbb{R}^{J_0} \times \mathbb{R}^{J_1} : \|p_0\| \leqslant 1,\ \|q_0\| \leqslant 1,\ \|q_1\| \leqslant 1\}$, whose bounds are normalized for convenience, and could be replaced by any positive values.

Throughout, $\Omega_k := \cup_{s\in S_k} \{s\} \times P_s^k$ is given, for each agent $k \in K$, summing up her final uncertainty at $t = 0$, unless otherwise stated. The collection $(\Omega_k)$ is an anticipation structure, along the following Definition, and we let $\underline{\boldsymbol{\Omega}} := \cap_{k\in K} \Omega_k$. We henceforth refer to $\Omega := S \times P$ as the forecast set.

**Definition 1** *An anticipation set is a closed subset of $\Omega$. An anticipation structure, whose set is denoted by $\mathcal{AS}$, is a collection of anticipation sets, $(\widetilde{\Omega}_k)$, such that:*

*(i)* $\forall s \in \underline{\mathbf{S}}$, $(\{s\} \times P) \cap (\cap_{k\in K} \widetilde{\Omega}_k) \neq \varnothing$.

*Given $(\widetilde{\Omega}_k) \in \mathcal{AS}$, an anticipation structure, $(\widetilde{\Omega}'_k) \in \mathcal{AS}$, which is smaller, for the inclusion relation, than $(\widetilde{\Omega}_k)$, is called a refinement of $(\widetilde{\Omega}_k)$, and denoted $(\widetilde{\Omega}'_k) \leq (\widetilde{\Omega}_k)$. A belief is probability distibution over $(\Omega, \mathcal{B}(\Omega))$, whose support is an anticipation set. A collection of beliefs, $(\widetilde{\pi}_k)$, whose supports define an anticipation structure, say $(\widetilde{\Omega}_k) \in \mathcal{AS}$, is called a structure of beliefs, said to support $(\widetilde{\Omega}_k)$, and denoted by $(\widetilde{\pi}_k) \in \Pi_{(\widetilde{\Omega}_k)}$. We let $\mathcal{SB}$ be the set of structures of beliefs and $\Pi^*_{(\widetilde{\Omega}_k)} := \cup_{(\widetilde{\Omega}'_k)\leq(\widetilde{\Omega}_k)} \Pi_{(\widetilde{\Omega}'_k)}$.*

so that its payoffs in such a service, $h \in H$, is always counted zero.

Henceforth, a structure, $(\pi_k) \in \Pi_{(\Omega_k)}$, is given, assumed to represent agents' beliefs at $t = 0$ and always referred to, unless stated otherwise. Non restrictively, along De Boisdeffre (2016), we also assume that the structure $(\Omega_k) \in \mathcal{AS}$ grants no agent an arbitrage opportunity on the financial market of the $\#J_0$ assets.

For every price system, $p := (p_s) \in P^{\underline{\mathbf{S}}}$, we let $V(p)$ be the $\underline{\mathbf{S}} \times J_0$ matrix, whose generic row is $V(p, s) := V(s, p_s)$ (for $s \in \underline{\mathbf{S}}$), and $< V(p) >$ be its span. We let $\mathcal{V}$ be the set of continuous payoff maps, $V' : \Omega \to \mathbb{R}^{J_0}$ defined as $V$, above, and equiped with the same notations. For every $\lambda \in \mathbb{R}_{++}$, we let $\mathcal{V}_\lambda := \{V' \in \mathcal{V} : \|V'(\omega) - V(\omega)\| \leqslant \lambda, \ \forall \omega \in \Omega\}$. We recall the following properties, from De Boisdeffre (2017 a).

**Claim 1** *Let* $\Lambda := \{\widetilde{V} \in \mathcal{V} : rank\ \widetilde{V}(p) = \#J_0, \ \forall p \in P\}$ *and* $\widetilde{V} \in \Lambda$ *be given.*
*The following Assertions hold:*
*(i) the set* $\Lambda$ *is open and everywhere dense in* $\mathcal{V}$*;*
*(ii)* $\nexists (z_k) \in (\mathbb{R}^{J_0})^K \backslash \{0\}, \ \nexists (\widetilde{\Omega}_k) \in \mathcal{AS} : \sum_{k \in K} \ z_k = 0$ *and* $\widetilde{V}(\omega_k) \cdot z_k \geqslant 0, \ \forall (k, \omega_k) \in K \times \widetilde{\Omega}_k$*.*

Given the anticipation structure, $(\Omega_k)$, the generic consumer, $i \in I$, forms her consumption plans within a subset, $X_i^o$, of the set, $C(\Omega'_i, \mathbb{R}_+^H)$, of continuous mappings from $\Omega'_i := \{0\} \cup \Omega_i$ to $\mathbb{R}_+^H$, where $\omega = 0$ denotes the non-random forecast at $t = 0$, that is, the pair of the non-random state, $s = 0$, and the observed price, $p_0 \in \mathbb{R}_+^H$. The inclusion $X_i^o \subset C(\Omega'_i, \mathbb{R}_+^H)$ is typically strict, for some goods and services, $h \in H$, are used as inputs by producers only, and not consumed (hence, zero components of consumptions). A consumption plan, $x \in X_i^o$, is a map relating continuously every forecast, $\omega \in \Omega'_i$, to a consumption decision, $x_\omega \in \mathbb{R}_+^H$, which is certain, if $\omega = 0$, and conditional on the realization of $\omega \in \Omega_i$, otherwise. Similarly, each producer, $j \in J$, elects a production plan within a production set, $Y_j^o \subset (\mathbb{R}^H)^{S'_j}$, representing her technology constraints. Differently from consumers', these sets, $Y_j^o$, do not depend on forecasts, and simply represent input-output technologically feasible combinations.

### 2.2 The producer's behaviour

Hereafter, a generic producer, $j \in J$, is given, with her belief, $\pi_j$, supporting $\Omega_j$.

Agent $j$ has a production set, $Y_j^o \subset (\mathbb{R}^H)^{S'_j}$, representing her technology constraints. It consists of all feasible input-output bundles, $y_s \in \mathbb{R}^H$, in every state, $s \in S'_j$, whose components are positive, if $h$ is an output, and negative, if used as an input. Many goods and services are not used or produced, so appear as zero components of the production plan, $y \in Y_j^o$. If production demands time, the inputs will typically be used at $t = 0$, and outputs be produced at $t = 1$. Standard assumptions on production sets are as follows, having a clear economic meaning:

***Assumption A1,*** $\forall j \in J,\ Y_j^o$ *is closed and convex*;

***Assumption A2,*** $\forall j \in J,\ Y_j^o \cap (\mathbb{R}_+^H)^{S'_j} = \{0\}$;

***Assumption A3,*** $\forall (j, \omega) \in J \times (\mathbb{R}_+^H)^{S'_j},\ (\{\omega\} + Y_j^o) \cap (\mathbb{R}_+^H)^{S'_j}$ *is bounded.*

If producers meet the above assumptions, they are said to be "standard", as we henceforth assume they are. As a classical result, their technologies have non-increasing returns to scale, consistently with competition. From Assumption *A3* and the limited quantity of inputs and endowments in the economy, production is bounded, that is, the production set, $Y_j^o$, can be assumed to be convex and compact.

The producer has subjective a discount factor, $\gamma_j \in [0, 1]$, of time, along which, at the observed first period price, $p_0 \in \mathbb{R}^H$, her discounted value of the expected profits of a production plan, $y := (y_s) \in Y_j^o$, is $p_0 \cdot y_0 + \gamma_j \int_{\omega := (s, p_s) \in \Omega_j} (p_s \cdot y_s) d\pi_j(\omega)$.

As standard, the producer is allowed to trade unrestrictively on assets. She is not on equities, which eventually belong to consumers. This, we could show, is non-restrictive because consumers - who are the eventual owners of corporations -

are free to exchange their corporate shares on stock markets. Typically, a producer would borrow at $t=0$ on the financial market to start her business. Yet, she is not allowed, or does not allow herself, to bankruptcy in any future state. This restriction is referred to as limited liability. An alternative setting would let agents own shares of each firm and be unrestrictively liable for potential losses up to their shares. Whatever the type of ownership, at the observed price, $\omega_0 := (p_0, (q_0, q_1)) \in P_0$, the producer's budget set is defined as follows:

$$B_j(\omega_0) := \{(y, z_0) \in Y_j^o \times \mathbb{R}^{J_0} : p_0 \cdot y_0 - q_0 \cdot z_0 \geqslant 0 \; and \; p_s \cdot y_s + V(\omega) \cdot z_0 \geqslant 0, \forall \omega := (s, p_s) \in \Omega_j\}.$$

This budget set is never empty (from Assumption *A2*). The producer has an objective function, $\Pi_j$, called profit, or returns' present value of her strategy, namely, for every $\omega_0 := (p_0, (q_0, q_1)) \in P_0$ and every $(y, z_0) \in B_j(\omega_0)$:

$$\Pi_j(\omega_0, (y, z_0)) = (p_0 \cdot y_0 - q_0 \cdot z_0) + \gamma_j \int_{\omega := (s, p_s) \in \Omega_j} (p_s \cdot y_s + V(\omega) \cdot z_0) d\pi_j(\omega).$$

Her behaviour is to maximise her profit in the budget set. The producer chooses (given $\omega_0 := (p_0, (q_0, q_1)) \in P_0$) one strategy $(y_j, z_{0j}) \in B_j(\omega_0)$, henceforth, set as given for all agents. This strategy results in the endogeneous yields, $r_{j0}(\omega_0, (y_j, z_{0j})) := (p_0 \cdot y_{0j} - q_0 \cdot z_{0j})$, at $t=0$, $r_{j\omega}(y_j, z_{0j}) := (p_s \cdot y_{js} + V(\omega) \cdot z_{0j})$, for all $\omega := (s, p_s) \in \Omega_j$, and $r_{j\omega}(y_j, z_{0j}) := 0$, for all $\omega \in \Omega \backslash \Omega_j$.

### 2.2 Consumers' behaviour and the concept of equilibrium,

Hereafter, a consumer $i \in I$ is given, with her anticipation set, $\Omega_i$, and belief, $\pi_i$.

The consumer receives an endowment, $e_i := (e_{is})$, granting her the conditional bundles of goods and services, $e_{i0} \in \mathbb{R}_+^H$ at $t=0$, and $e_{is} \in \mathbb{R}_+^H$, in each expected state, $s \in S_i$, if it prevails. Any good or service, $h \in H$, in any state, in which the economy is endowed is useful without satiation to at least one agent, $k \in K$. The consumer's endowment in services consists in an amount of labour with certain skills, called workforce, that she may offer to producers. The agent consumes leisure if she does not offer her full workforce.

The agent's consumption set, $X_i^o \subset (\mathbb{R}_+^H)^{\Omega_i'}$, is the subset of continuous mappings, $x : \Omega_i' \to \mathbb{R}_+^H$, which relate every forecast, $\omega \in \Omega_i'$, to a consumption decision, $x_\omega \in \mathbb{R}_+^H$ (certain at $t = 0$, and conditional, at $t = 1$), whose components on intermediary goods and raw materials, only used by firms, are zero.

In addition to their endowments, consumers may receive dividends. Indeed, each firm, $j \in J$, belongs to consumers, either exclusively, or partly, as partners or shareholders. Each agent, $i \in I$, detains initial (possibly zero) shares, $\overline{z}_{1i} := (\overline{z}_{1i}^j) \in [0,1]^{J_1}$, of each corporation, $j \in J_1$, and $\overline{z}_{2i} := (\overline{z}_{2i}^j) \in [0,1]^{J_2}$, of other companies, which satisfy $\sum_{i \in I} \overline{z}_{1i} = (1,1,...,1) \in \mathbb{R}^{J_1}$ and $\sum_{i \in I} \overline{z}_{2i} = (1,1,...,1) \in \mathbb{R}^{J_2}$. Most of these shares (components) should be zero. We recall ownership breaks down into three categories:

*** sole proprietorship**

A company, $j \in J_2$, is owned by one person, $i \in I$ (i.e., $\overline{z}_{2i}^j = 1$), so that $\pi_i = \pi_j$. The company may be uneasy to sell is assumed to be kept across periods.

*** partnership**

It occurs when a limited number of partners, $i \in I_j \subset I$, have agreed to create a joint venture, $j \in J_2$, and on the shares, $\overline{z}_{2i}^j > 0$, of each member. The latter are such that $\sum_{i \in I_j} \overline{z}_{2i}^j = 1$. Partners may also have difficulties in retrading their shares, which they keep at both periods.

To the difference of sole owners, partners may have different assessments of future income streams, resulting in potential management disagreements. Conflicts can often be resolved by side payments, whose study is beyond our scope. In any case, joint ventures only create if partners have reached a managerial agreement.

Partners would be expected to share their information so that $\Omega_i = \Omega_j$, for every $i \in I_j$. However, the model does not impose this. If partners do not share the same beliefs, the shareholder, $i \in I_j$, of the firm, $j \in J_2$, expects to receive her share of profits in every forecast, $\omega \in \Omega_i \cap \Omega_j$, common with the firm.

*** corporations**

Corporations', $j \in J_1$, shares may be exchanged on the stock market by all consumers, deciding to keep or change their initial shares, $(\overline{z}_{1i})$, for new ones $(z_{1i})$, along their perceived interests, at a market price, $q_1 \in \mathbb{R}^{J_1}$. Speculation is amongst the investors' motives. Corporations are run by an appointed manager and owned by private shareholders, (possibly) meeting in boards and always free to exchange participations on the stock market. Shareholders are assumed to know their corporations' strategies, hence, their endogenous yields.

To the difference of assets $(j \in J_0)$, corporations $(j \in J_1)$ have endogenous yields (as defined from the above firm strategy), and their purchase and sale are bounded in practice. Indeed, corporations are physical units, which cannot be bought or sold short an unlimited number of times. Transactions are thus bounded. We assume, w.l.o.g. on the bounds, that a corporation cannot be sold short and cannot be bought more than one time by any agent. Hence, corporations' porfolio set is $[0,1]^{J_1}$.

We now present the agent's behaviour and the concept of equilibrium. Given the observed prices, $\omega_0 := (p_0, (q_0, q_1)) \in P_0$, and production strategies, $[(y_j, z_{0j})] \in \times_{j \in J} B_j(\omega_0)$, that producers have elected, the consumer's budget set is:

$B_i(\omega_0,[(y_j,z_{0j})]) := \{(x:=(x_\omega), z:=(z_0,z_1)) \in X_i^o \times \mathbb{R}^{J_0} \times [0,1]^{J_1}:$

$p_0{\cdot}(x_0-e_{i0}) \leqslant -q_0{\cdot}z_0 - q_1{\cdot}(z_1-\overline{z}_{1i}) + \sum_{j\in J_1}(z_1^j-\overline{z}_{1i}^j)\ r_{j0}(\omega_0,(y_j,z_{0j})) + \sum_{j\in J_2}\overline{z}_{2i}^j\ r_{j0}(\omega_0,(y_j,z_{0j}))$

*and* $p_s{\cdot}(x_s-e_{is}) \leqslant V(\omega){\cdot}z_0 + \sum_{j\in J_1}(z_1^j-\overline{z}_{1i}^j)\ r_{j\omega}(y_j,z_{0j}) + \sum_{j\in J_2}\overline{z}_{2i}^j\ r_{j\omega}(y_j,z_{0j}),\ \forall\omega := (s,p_s)\in\Omega_i\}.$

The consumer's welfare is measured, ex post, by a continuous utility index, $u_i : \mathbb{R}_+^{2H} \to \mathbb{R}_+$, over her consumptions at both dates. Ex ante, her preferences are represented by the V.N.M. utility function: $x \in X_i^o \mapsto U_i(x) := \int_{\omega\in\Omega_i} u_i(x_0,x_\omega)d\pi_i(\omega)$.

In the above economy, $\mathcal{E}_{(V,(\pi_k))} = \{(I,J_0,J_1,J_2,H), V, (\Omega_k,\pi_k)_{k\in K}, (Y_j^o)_{j\in J_1\cup J_2}, (X_i^o,e_i,u_i)_{i\in I}\}$, agents optimise their objective functions in budget sets. So the equilibrium concept:

**Definition 2** *A collection of prices,* $\omega_0 := (p_0, q := (q_0,q_1)) \in P_0$ *and* $p := (p_s) \in P^{\underline{\mathbf{S}}}$, *and strategies,* $[(y_j,z_{0j})] \in \times_{j\in J}\ B_j(\omega_0)$ *and* $[(x_i, z_i := (z_{0i},z_{1i}))] \in \times_{i\in I}\ B_i(\omega_0,[(y_j,z_{0j})])$, *defines a (sequential) equilibrium of the economy,* $\mathcal{E}_{(V,(\pi_k))}$, *or correct foresight equilibrium (C.F.E.), if the following conditions hold:*

*(a)* $\forall s\in\underline{\mathbf{S}},\ \forall k\in K,\ (s,p_s)\in\Omega_k$;

*(b)* $\forall j\in J,\ (y_j,z_{0j}) \in \arg\max \Pi_j(\omega_0,(y,z))$ *for* $(y,z)\in B_j(\omega_0)$;

*(c)* $\forall i\in I,\ (x_i,z_i) \in \arg\max U_i(x))$ *for* $(x,z)\in B_i(\omega_0,[(y_j,z_{0j})])$;

*(d)* $\sum_{i\in I}(x_{i(s,p_s)}-e_{is}) = \sum_{j\in J}\ y_{js},\ \forall s\in\underline{\mathbf{S}}'$;

*(e)* $\sum_{k\in K} z_{0k} = 0$ *and* $\sum_{i\in I} z_{1i} = \sum_{i\in I}\overline{z}_{1i}$.

*Under above conditions, each forecast,* $(s,p_s)\in\underline{\mathbf{S}}\times P$, *is said to support equilibrium.*

**Definition 3** *Let* $\widetilde{V}\in\mathcal{V}$, $(\widetilde{\Omega}_k)\in\mathcal{AS}$ *and* $(\widetilde{\pi}_k)\in\Pi_{(\widetilde{\Omega}_k)}$ *be given. An equilibrium of the economy* $\mathcal{E}_{(\widetilde{V},(\widetilde{\pi}_k))}$, *and its supporting forecasts, are defined the same as in Definition 2, after replacing the payoff map,* $V$, *by* $\widetilde{V}$, *the anticipation sets,* $(\Omega_k)$, *by* $(\widetilde{\Omega}_k)$, *and beliefs,* $(\pi_k)$, *by* $(\widetilde{\pi}_k)$, *in all consumption sets, budget sets, profit and utility functions.*

The economy is called standard under conditions ***A1*** to ***A3*** and the following:

- ***Assumption A4*** *(strong survival):* $\forall i \in I,\ e_i \in (\mathbb{R}^H_{++})^{S'_i}$;

- ***Assumption A5****: for each* $i \in I$*,* $u_i$ *is continuous, strictly concave and increasing:* $[(x, y, x', y') \in (\mathbb{R}^H_+)^4,\ (x, y) \leqslant (x', y'),\ (x, y) \neq (x', y')] \Rightarrow [u_i(x', y') > u_i(x, y)]$.

# 3 The existence theorem and proof

The following Theorem shows that the existence of equilibrium is related to an incompressible uncertainty resulting from the fact that agents have private characteristics and beliefs and no function to forecast prices. Admissible forecasts can only be inferred from observing past prices.

### 3.1 The minimum uncertainty set

**Definition 4** *The minimum uncertainty set is the (non-empty) set,* $\Delta \subset \underline{\mathbf{S}} \times P$*, of forecasts, which support the equilibria of an economy,* $\mathcal{E}_{(V,(\widetilde{\pi}_k))}$*, for some beliefs,* $(\widetilde{\pi}_k) \in \mathcal{SB}$*, today.*

**Definition 5** *Given* $n \in \mathbb{N}$*, the* $n$*-uncertainty set is the (non-empty) set,* $\Delta_n \subset \underline{\mathbf{S}} \times P$*, of forecasts, which support the equilibria of the economies,* $\mathcal{E}_{(\widetilde{V},(\widetilde{\pi}_k))}$*, defined for all payoff maps,* $\widetilde{V} \in \mathcal{V}_{1/n}$*, and all beliefs,* $(\widetilde{\pi}_k) \in \mathcal{SB}$*.*

**Lemma 1** *In a standard economy,* $\mathcal{E}$*, there exists* $\varepsilon \in ]0, 1[$*, such that:*
$\forall n \in \mathbb{N},\ \Delta \subset \Delta_{n+1} \subset \Delta_n \subset \Delta_1 \subset \underline{\mathbf{S}} \times [\varepsilon, 1]^H$.

**Proof** Lemma 1 is a direct corollary of De Boisdeffre's (2017 a) Lemmata 1. □

**Theorem 1** *If* $\Delta \subset \underline{\mathbf{\Omega}} := \cap\, \Omega_k$*, a standard economy,* $\mathcal{E}_{(V,(\pi_i))}$*, has an equilibrium.*

Henceforth, we assume that the economy, $\mathcal{E}_{(V,(\pi_i))}$, is standard. We construct a sequence of auxiliary finite economies, tending to the initial one. All finite economies admit equilibria, whose sequence yields a C.F.E. Hereafter, we provisionally assume that $\Delta^* := \lim_{n\to\infty} \searrow \overline{\Delta_n} \subset \underline{\mathbf{\Omega}}$ (instead of $\Delta \subset \underline{\mathbf{\Omega}}$, along Theorem 1). The proof is similar to De Boisdeffre's (2017 b) pure-exchange, whose steps are now recalled.

### 3.2 Finite partitions of agents' anticipation sets

- Let $(k,n) \in K \times \mathbb{N}$ be given. We define a partition, $\mathcal{P}_k^n = \{\Omega_{(k,n)}^m\}_{1\leqslant m\leqslant M_{(k,n)}}$, of $\Omega_k$, such that $\pi_k(\Omega_{(k,n)}^m) > 0$, for each $m \leqslant M_{(k,n)}$.
- In each set $\Omega_{(k,n)}^m$ (for $m \leqslant M_{(k,n)}$), we select exactly one interior element, $\omega_{(k,n)}^m$, forming the set, $\Omega_k^n := \{\omega_{(k,n)}^m\}_{1\leqslant m\leqslant M_{(k,n)}}$.
- We define mappings, $\pi_k^n : \Omega_k^n \to \mathbb{R}_{++}$, by $\pi_k^n(\omega_{(k,n)}^m) = \pi_k(\Omega_{(i,n)}^m)$ and $\Phi_k^n : \Omega_k \to \Omega_k^n$, by its restrictions, $\Phi_{k\ /\ \Omega_{(k,n)}^m}^n(\omega) = \omega_{(k,n)}^m$, for each $m \leqslant M_{(k,n)}$.

**Lemma 2** *For each $k \in K$, we may choose the above defined sequences, $\{\mathcal{P}_k^n\}_{n\in\mathbb{N}}$, $\{\Omega_k^n\}_{n\in\mathbb{N}}$ and $\{\Phi_k^n\}_{n\in\mathbb{N}}$, such that:*

$(i)$ *for every $n \in \mathbb{N}$, $\Omega_k^n \subset \Omega_k^{n+1}$ and $\mathcal{P}_k^{n+1}$ is finer than $\mathcal{P}_k^n$;*

$(ii)$ $\Omega_k = \overline{\lim_{n\to\infty} \nearrow \Omega_k^n} = \overline{\cup_{n\in\mathbb{N}}\Omega_k^n}$, *that is, $\cup_{n\in\mathbb{N}}\Omega_k^n$ is everywhere dense in $\Omega_k$;*

$(iii)$ *for every $\omega \in \Omega_k$, $\omega = \lim_{n\to\infty} \Phi_k^n(\omega)$, and $\Phi_k^n(\omega)$ converges uniformly to $\omega$.*

**Proof** The proof is the same as Lemma 2's in De Boisdeffre (20017 b). □

### 3.3 The auxiliary economies, $\mathcal{E}^n$

Given $n \in \mathbb{N}$, we define an economy, $\mathcal{E}^n$, which is of the type $\mathcal{E}_{(V_n,(\pi_k^n))}$, for some map $V_n \in \Lambda \cap \mathcal{V}_{1/n} \neq \varnothing$, from Claim 1, and some beliefs, $(\widetilde{\pi}_k) \in \mathcal{SB}$, with a slight abuse in the anticipation structure, $(\Theta_k^n)$. This economy, $\mathcal{E}^n$, is defined as follows:

- for each $k \in K$, we let $\Omega_k^{*n} := \{k\} \times \Omega_k^n$ and $\Theta_k^n := \underline{\mathbf{S}} \cup \Omega_k^{*n}$ define an information structure, $(\Theta_k^n)$, of a formal state space, $\Theta^n := \cup_{k\in K}\Theta_k^n$.

- for each $k \in K$, we let $\pi_k^{*n}$ be the probability on $\Theta_k^n$ defined by $\pi_k^{*n}((k,s,p)) := (1 - 1/2n\#\underline{\mathbf{S}})\pi_k^n((s,p))$, for every $(s,p) \in \Omega_k^n$, and $\pi_k^{*n}(s) := 1/2n\#\underline{\mathbf{S}}$, for every $s \in \underline{\mathbf{S}}$.

- In each (realizable) state $s \in \underline{\mathbf{S}}$, the generic $k^{th}$ agent is assumed to anticipate with perfect foresight the spot price to prevail.

- In each (purely formal) state $(k,s,p) \in \Omega_k^{*n}$, the agent has an idiosyncratic certainty that price $p \in P$ will prevail.

- The map, $V_n \in \Lambda \cap \mathcal{V}_{1/n}$, is chosen arbitrarily and set as given.

Let the observed prices, $\omega_0^n := (p_0^n, (q_0^n, q_1^n)) \in P_0$, and perfectly anticipated prices, $p^n := (p_s^n) \in P^{\underline{\mathbf{S}}}$, be given.

The generic $j^{th}$ producer's production set and discount factor are the above $Y_j^o$ and $\gamma_j$, and her budget set and profit function are, respectively:

$$B_j^n(\omega_0^n, p^n) := \{(y, z_0) \in Y_j^o \times \mathbb{R}^{J_0} : p_0^n \cdot y_0 - q_0^n \cdot z_0 \geqslant 0,$$

$$p_s^n \cdot y_s + V_n(s, p_s^n) \cdot z_0 \geqslant 0,\ \forall s \in \underline{\mathbf{S}}, \quad \textit{and} \quad p_s \cdot y_s + V_n(\omega) \cdot z_0 \geqslant 0, \forall \omega := (s, p_s) \in \Omega_j^n\},$$

$$(y, z_0) \in B_j^n(\omega_0^n, p^n) \longmapsto \Pi_j^n(\omega_0^n, p^n, (y, z_0)) = (p_0^n \cdot y_0 - q_0^n \cdot z_0) +$$

$$\sum_{\theta := (j,s,p_s) \in \Omega_j^{*n}} \gamma_j\ \pi_j^{*n}(\theta)\ [p_s \cdot y_s + V_n(s, p_s) \cdot z_0] + \sum_{s \in \underline{\mathbf{S}}} \gamma_j\ \pi_j^{*n}(s)\ [p_s^n \cdot y_s + V_n(s, p_s^n) \cdot z_0].$$

She elects one strategy, $(y_j^n, z_{0j}^n) \in B_j^n(\omega_0^n, p^n)$, henceforth given, whose yields are: $r_{j0}^n := (p_0^n \cdot y_{0j}^n - q_0^n \cdot z_{0j}^n)$, at $t = 0$, $r_{js}^n := (p_s^n \cdot y_{js}^n + V_n(s, p_s^n) \cdot z_{0j}^n)$, for every $s \in \underline{\mathbf{S}}$, $r_{j\theta}^n := (p_s \cdot y_{js}^n + V_n(s, p_s) \cdot z_{0j}^n)$, for every $\theta := (j, s, p_s) \in \Omega_j^{*n}$, and $r_{j\theta}^n := 0$ for $\theta \in \Theta^n \backslash \Theta_j^n$.

The generic $i^{th}$ consumer's consumption set is $X_i^n \subset \mathbb{R}_+^{\Theta_i'^n}$, where we let $\Theta_i'^n := \{0\} \cup \Theta_i^n$. The vectors, $x := (x_\theta) \in X_i^n$, have zero components in non-consumption goods. The agent's budget set is:

$B_i^n(\omega_0^n, p^n, [(y_j^n, z_{0j}^n)]) := \{(x,\ z := (z_0, z_1)) \in X_i^n \times \mathbb{R}^{J_0} \times [0,1]^{J_1} :$

$p_0^n{\cdot}(x_0 - e_{i0}) \leqslant -q_0^n{\cdot}z_0 - q_1^n{\cdot}(z_1 - \overline{z}_{1i}) + \sum_{j\in J_1}(z_1^j - \overline{z}_{1i}^j)\ r_{j0}^n + \sum_{j\in J_2} \overline{z}_{2i}^j\ r_{j0}^n$

$p_s^n{\cdot}(x_s - e_{is}) \leqslant V_n(s, p_s^n){\cdot}z_0 + \sum_{j\in J_1}(z_1^j - \overline{z}_{1i}^j)\ r_{js}^n + \sum_{j\in J_2} \overline{z}_{2i}^j\ r_{js}^n,\ \forall s \in \underline{\mathbf{S}}$

$p_s{\cdot}(x_\theta - e_{is}) \leqslant V_n(s, p_s){\cdot}z_0 + \sum_{j\in J_1}(z_1^j - \overline{z}_{1i}^j)\ r_{j\theta}^n + \sum_{j\in J_2} \overline{z}_{2i}^j\ r_{j\theta}^n,\ \forall \theta := (j, s, p_s) \in \Omega_i^{*n}\}.$

The agent's utility function is: $u_i^n : x \in X_i^n \mapsto\ u_i^n(x) := \sum_{\theta\in\Theta_i^n} \pi_i^{*n}(\theta)\ u_i(x_0, x_\theta).$

**Definition 6** *The collection of prices, $\omega_0^n \in P_0$ and $p^n \in P^{\underline{\mathbf{S}}}$, and of agents' strategies, $[(y_j^n, z_{0j}^n)] \in \times_{j\in J}\ B_j^n(\omega_0^n, p^n)$ and $[(x_i^n, z_i^n := (z_{0i}^n, z_{1i}^n))] \in \times_{i\in I}\ B_i^n(\omega_0^n, p^n, [(y_j^n, z_{0j}^n)])$, defines an equilibrium of the economy, $\mathcal{E}^n$, if the following conditions hold:*

*(a) $\forall j \in J,\ (y_j^n, z_{0j}^n) \in \arg\max \Pi_j^n(\omega_0^n, p^n, (y, z))$ for $(y, z) \in B_j^n(\omega_0^n, p^n)$;*

*(b) $\forall i \in I,\ (x_i^n, z_i^n) \in \arg\max u_i^n(x))$ for $(x, z) \in B_i^n(\omega_0, [(y_j, z_{0j})])$;*

*(c) $\sum_{i\in I}(x_{is}^n - e_{is}) = \sum_{j\in J}\ y_{js}^n,\ \forall s \in \underline{\mathbf{S}}'$;*

*(d) $\sum_{k\in K} z_{0k} = 0$ and $\sum_{i\in I} z_{1i} = \sum_{i\in I} \overline{z}_{1i}$.*

From De Boisdeffre's (2017 c) Theorem 1, the economy, $\mathcal{E}^n$, has an equilibrium, $\mathcal{C}^n := ((\omega_0^n, p^n), [(x_i^n, z_i^n), [(x_j^n, z_{0j}^n)])$, henceforth given, with the following properties:

**Lemma 3** *Let the sequence $\{\mathcal{C}^n\}_{n\in\mathbb{N}}$, be given from above. The following holds:*

*(i) the sequence, $\{(\omega_0^n, p^n)\}_{n\in\mathbb{N}}$, may be assumed to converge, say to $(\omega_0^*, p^*) \in P_0 \times \overline{P^{\underline{\mathbf{S}}}}$, such that $\{(s, p_s^*)\}_{s\in\underline{\mathbf{S}}} \subset \Delta^*$;*

*(ii) the sequences $\{(x_{is}^n)_{s\in\underline{\mathbf{S}}'}\}$, $\{(y_j^n)\}$, $\{(z_{0k}^n)\}$ and $\{(z_{1i}^n)\}$ may be assumed to converge, say to $(x_{is}^*)_{s\in\underline{\mathbf{S}}'} \in (\mathbb{R}_+^H)^{\underline{\mathbf{S}}'}$, $(y_j^*) \in \times_{j\in J} Y_j^o$, $(z_{0k}^*) \in (\mathbb{R}^{J_0})^K$, $(z_{1i}^*) \in (\mathbb{R}^{J_1})^I$, such that $\sum_{i\in I}(x_{is}^* - e_{is})_{s\in\underline{\mathbf{S}}'} = \sum_{j\in J}(y_{js}^*)_{s\in\underline{\mathbf{S}}'}$, $\sum_{k\in K} z_{0k}^* = 0$ and $\sum_{i\in I} z_{1i}^* = \sum_{i\in I} \overline{z}_{1i}$.*

**Lemma 4** *Let $B_i(\omega, z) = \{x \in \mathbb{R}_+^H : p{\cdot}(x - e_{is}) \leqslant V(\omega){\cdot}z_0 + \sum_{j\in J_1}(z_{1i}^{*j} - \overline{z}_{1i}^j)\ r_{j\omega}(y_j^*, z_{0j}^*) + \sum_{j\in J_2} \overline{z}_{2i}^j\ r_{j\omega}(y_j^*, z_{0j}^*)\}$, for every $(i, z := (z_0, z_1), \omega := (s, p)) \in I{\times}\mathbb{R}^{J_0}{\times}\mathbb{R}^{J_1}{\times}\Omega_i$, be given sets. Denote by $\omega_s^* := (s, p_s^*)$, and $x_{i\omega_s^*}^* := x_{is}^*$, for each $(i, s) \in I{\times}\underline{\mathbf{S}}$, the limits of*

*Lemma 3. Then, the following Assertions hold, for each* $(i,j) \in I \times J$:

$(i)$ *for every* $s \in \underline{\mathbf{S}}$, $\{x^*_{i\omega^*_s}\} = \arg\max\ u_i(x^*_{i0}, x)$, *for* $x \in B_i(\omega^*_s, z^*_i)$*;*

$(ii)$ *the correspondence* $\omega \in \Omega_i \mapsto \arg\max\ u_i(x^*_{i0}, x)$, *for* $x \in B_i(\omega, z^*_i)$, *is a continuous mapping, whose embedding,* $x^*_i : \omega \in \Omega'_i \mapsto x^*_{i\omega}$, *defines a consumption plan;*

$(iii)$ $U_i(x^*_i) = \lim_{n\to\infty} u^n_i(x^n_i)$*;*

$(iv)$ $\Pi_j(\omega^*_0, (y^*_j, z^*_{0j})) = \lim_{n\to\infty}\ \Pi^n_j(\omega^n_0, (y^n_j, z^n_{0j}))$.

**Proofs** Recalling our comments following Assumption *A3*, above, all production sets may be assumed to be convex and compact. It follows that $\{(y^n_j)\}$ may be assumed to converge to some $(y^*_j) \in \times_{j\in J} Y^o_j$. Up to the change in total supply, demand and consumer's income, due to producers, and in the number of portfolios and traders, the proofs of Lemma 3 & Lemma 4-$(i)$-$(ii)$-$(iii)$, are identical to those of Lemma 3 and 4 in De Boisdeffre (2017 b), to which we refer the reader.

Lemma 4-$(iv)$ We recall that, for each $n \in \mathbb{N}$, $\Pi^n_j(\omega^n_0, p^n, (y^n_j, z^n_{0j})) = (p^n_0 \cdot y^n_{j0} - q^n_0 \cdot z^n_{0j}) + \sum_{\theta:=(j,s,p_s)\in\Omega^{*n}_j} \gamma_j\ \pi^{*n}_j(\theta)\ [p_s \cdot y^n_{js} + V_n(s,p_s)\cdot z^n_{0j}] + \sum_{s\in\underline{\mathbf{S}}} \gamma_j\ \pi^{*n}_j(s)\ [p^n_s \cdot y^n_{js} + V_n(s,p^n_s)\cdot z^n_{0j}]$, whereas $\Pi_j(\omega^*_0, (y^*_j, z^*_{0j})) = (p^*_0 \cdot y^*_{j0} - q^*_0 \cdot z^*_{0j}) + \gamma_j \int_{\omega:=(s,p_s)\in\Omega_j} (p^*_s \cdot y^*_s + V(\omega)\cdot z^*_{0j}) d\pi_j(\omega)$. Then, the proof of Lemma 4-$(iv)$ is immediate from Lemma 3, the above definitions and the continuity of the scalar product, and is left to the reader. $\square$

### 3.4 An equilibrium of the initial economy

Theorem 1 follows from Claim 2.

**Claim 2** *The collection,* $\mathcal{C} := \{(\omega^*_s), (x^*_i), (y^*_j), (z^*_{0i}), (z^*_{0j}), (z^*_{1i})\}$, *of prices, forecasts, allocation and portfolios of Lemmas 3-4, defines a CFE of the economy* $\mathcal{E}_{(V,(\pi_i))}$.

**Proof** Let $j \in J$ be given. We show, first, that $(y^*_j, z^*_{0j})$ maximizes the producer's profit in the budget set, $B_j(\omega^*_0) := \{(y, z_0) \in Y^o_j \times \mathbb{R}^{J_0} : p^*_0 \cdot y_0 - q^*_0 \cdot z_0 \geqslant 0$ *and* $p_s \cdot y_s +$

$V(\omega) \cdot z_0 \geqslant 0, \forall \omega := (s, p_s) \in \Omega_j\}$. Assume, by contraposition, that there exist $\varepsilon > 0$ and $(y, z_0) \in B_j(\omega_0^*)$, such that:

$$(I) \qquad 2\varepsilon + \Pi_j(\omega_0^*, (y_j^*, z_{0j}^*)) < \Pi_j(\omega_0^*, (y, z_0)).$$

From Lemma 2 and 3 the continuity of the scalar product and the definition of budget sets, we may assume that there exist $N \in N$, such that $(y, z_0) \in B_j^n(\omega_0^n)$ for every $n \geqslant N$. Then, the definition of auxiliary equilibria yield, for every $n \geqslant N$:

$$(II) \qquad \Pi_j^n(\omega_0^n, (y, z_0)) \leqslant \Pi_j^n(\omega_0^n, (y_j^n, z_{0j}^n)).$$

From Lemma 4-$(iv)$, we let $n \geqslant N$ be such that:

$$(III) \qquad \Pi_j^n(\omega_0^n, (y_j^n, z_{0j}^n)) - \varepsilon \leqslant \Pi_j(\omega_0^*, (y_j^*, z_{0j}^*)).$$

$(I)$-$(II)$-$(III)$ yield: $\varepsilon + \Pi_j(\omega_0^*, (y_j^*, z_{0j}^*)) < \Pi_j(\omega_0^*, (y, z_0))$-$\varepsilon \leqslant \Pi_j^n(\omega_0^n, (y_j^n, z_{0j}^n))$-$\varepsilon \leqslant \Pi_j(\omega_0^*, (y_j^*, z_{0j}^*))$.

This contradiction proves that the strategies, $[(y_j^*, z_{0j}^*)] \in \times_{j \in J} B_j(\omega_0^*)$, are optimal for all producers. The collection, $\mathcal{C}$, of Claim 2 meets conditions $(d)$ and $(e)$ of Definition 2 of equilibrium, above, from Lemmas 3 and 4. It also meets conditions $(a)$ and $(c)$ of Definition 2. Up to the change in consumer's income, due to dividends, this part of the proof of Claim 2, above, is identical to that of Claim 1, in De Boisdeffre (2017 b), in a pure-exchange economy, to which we refer the reader. □

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